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RESEARCH PROJECT INITIATION

Date: 26 February 1974

Project Title: **Finite Element Analysis of Elastic-Plastic Three Dimensional Cracks**

Project No: **E-23-608**

Principal Investigator **Dr. S. Atluri**

Sponsor: **AFOSR; Arlington, Virginia**

Agreement Period: From 2/1/74 Until 1/31/75

Type Agreement: **Grant No. AFOSR 74-2667**

Amount: **\$35,000 AFOSR funds (E-23-608)**
9,862 GIT Contrib. (E-23-317)
\$44,862 TOTAL

Reports Required:

Final Scientific Report; Interim (Annual) Report if project extended beyond one year.

Sponsor Contact Person (s):

Technical Matters

William J. Walker
Program Manager
AFOSR (NA)
1400 Wilson Boulevard
Arlington, Virginia 22209

Contractual Matters

(Thru ORA)
Joan O. Marshall
Buyer
AFOSR (PMD)
1400 Wilson Boulevard
Arlington, Virginia 22209

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Post
off

Date: 4/12/78

Project Title: Finite Element Analysis of Elastic-Plastic Three Dimensional Cracks

Project No: E-23-608

Project Director: Dr. S. Atluri

Sponsor: Air Force Office of Scientific Research, Bolling AFB, D.C. 20332

Effective Termination Date: 1/31/78

Clearance of Accounting Charges: 1/31/78

Grant/Contract Closeout Actions Remaining:

Grant AFOSR 74-2667

- ☐ Final Invoice and Closing Documents
- ☒ Final Fiscal Report
- ☐ Final Report of Inventions
- ☒ Govt. Property Inventory & Related Certificate
- ☐ Classified Material Certificate
- ☐ Other

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Interim Report of Research on
"Finite-Element Analysis of Elastic-Plastic Three-Dimensional Cracks"

February 1, 1974 - February 1, 1975

Grant AFOSR 74-2667

Satya N. Atluri
School of Engineering Science and Mechanics
Georgia Institute of Technology
Atlanta, Georgia 30332

March 1975

Interim Report on AFOSR 74-2667:

Elastic-Plastic Analysis of Three Dimensional Cracks

1. Introduction

Reported here-in is a summary of research on Elastic-Plastic Analysis of Three-Dimensional Cracks supported by AFOSR Grant 74-2667 and performed during the period February 1, 1974 to January 31, 1975. It represents an elaboration of the brief progress report to Dr. W. J. Walker, sent in October 1974.

The research described has been performed at the School of Engineering Science and Mechanics, Georgia Institute of Technology, under the direction of Dr. Satya Atluri. Two Doctoral students, Mr. K. Kathiresan, and Mr. Wen-Hwa Chen have also been participating in the research, and their Doctoral theses are proposed to be written in the same field of research. Dr. Nakagaki who graduate from the University of Washington in Seattle, has joined in the research effort as a Post-Doctoral Research Associate, on December 2, 1974.

Dr. M. E. Raville, Director, School of Engineering Science and Mechanics, has provided immense support to make it possible to assemble the above research team. Dr. W. J. Walker of AFOSR, provided continued encouragement and has aided immensely in defining the ultimate practical objectives of this basic research.

The studies under the present grant included: (a) development of the two-dimensional finite element procedure to calculate stress intensity factors corresponding to general $1/r^\alpha$ stress singularities in composites;

(b) study of the convergence of the assumed displacement hybrid finite element procedure in fracture mechanics problems; (c) development of a three-dimensional finite element procedure to calculate the elastic combined mode stress-intensity factors K_I , K_{II} and K_{III} that vary along an arbitrarily curved three-dimensional crack front; (d) development of a two-dimensional finite element procedure for analysing plane problems of fracture mechanics involving large-scale yielding conditions under cyclic loading; and (e) extension of (d) to study the effects of crack opening and closure stresses in fatigue crack propagation.

A brief report on progress made on each of these efforts to date, and comments on further work are given below. A listing of publications that have resulted from the research is included.

2. Summary of Research Accomplishments

A. Fracture Mechanics of Composite Materials

The finite element procedure based on an assumed displacement hybrid finite element model has been refined and was made exhaustive to calculate the stress intensity factors for mixed mode behavior of arbitrary shaped curved cracks in plane stress and plane strain problems involving anisotropic, nonhomogeneous, but linearly elastic materials.

Part of this research effort has been coordinated with Dr. G. P. Sendeckyj so that the present results could be used by Dr. Sendeckyj in his AFFDL (Wright-Patterson AFB) program for the evaluation of composite material properties.

A summary of the types of problems that have been solved is as follows:

(a) Unsymmetric boundary conditions and configurations: An oblique edge

crack in a tension plate of aspect ratio $(L/b) = 2$. To calculate the mixed mode stress intensity factors in this isotropic case, a total of 42 quadrilateral elements (including from singular elements) with 228 degrees of freedom were used. The directly calculated stress intensity factors were within 1.5% of those theoretically obtained by using collocation methods by Bowie and Freese. In addition, a quarter circular crack centrally located in a tension plate of aspect ratio one, was solved. A total of 48 elements with 246 degrees of freedom were used. The results for both biaxial and uniaxial tension cases were within 2% of the theoretical predictions by Sih using a complex variable approach for an infinite sheet.

(b) Stress, displacement, or mixed conditions: The presently developed procedure is capable of handling all these boundary conditions if they actually occur in the given problem or arise out of symmetry conditions.

(c) Curved cracks: Such cases, which are analysed, are already discussed above. Because curved isoparametric quadrilateral elements were employed, the curved boundary of the crack need not be approximated by straight line segments.

(d) Material nonhomogeneity: A bimaterial tension plate, in plane stress or plane strain conditions, with a crack running perpendicular to the material interface was analysed. The nature of singularity in stresses is of $r^{-\frac{1}{2}}$ type at the two tips of the crack, whereas the stress singularity is of general $r^{-\alpha}$ type at the point of intersection of the crack and the material interface (where the value of α depends on the material properties). Thus, special elements containing $r^{-\alpha}$ singularities in stresses were used at the intersection of the crack and material interface. The finite-element results

were within 3% of the theoretical results previously obtained by Erdogan and Biricikoglu for an infinite sheet with a similar crack.

(e) Cracks in orthotropic sheets: A centrally cracked orthotropic tension plate of aspect ratio one was analysed. A total of only 24 elements with 52 degrees of freedom were employed. The results for various values of the ratios E_x/E_y agreed to within 2% of those obtained by Bowie and Reese who used a complex variable formulation. The procedure was also used to analyze two fracture test specimens for which analytical results do not exist. One solution is the doubly-edge-notched tension plate, with material principal directions oriented 0° - 90° or $\pm 45^\circ$ to the geometric axes of symmetry, and with varying crack lengths. The second solution is a three-point bend specimen with material principal directions oriented 0° - 90° to the geometric axes of symmetry.

(f) Cracks meeting a free surface: Single-edge-notched (SEN) tension plates with varying aspect ratios and varying crack lengths were analysed.

(g) Cracks unsymmetric with respect to axes of orthotropy: A doubly-edge-notched tension plate with material principal directions oriented at $\pm 15^\circ$ and $\pm 65^\circ$ to the geometric axes of symmetry were analysed. Also an oblique center crack in an orthotropic tension plate with material principal orthotropy directions being the same as the geomaterial-symmetry axes was analysed. All these problems for which no other theoretical solutions exist would be used for comparison by Dr. Sendekyj in this AFFDL program on experimental study of composite materials.

From the above it can be seen that the developed finite element procedure is very versatile in analysing an exhaustive and wide variety of plane problems of linear fracture mechanics.

This program, with the approval of Dr. W. J. Walker, has been released to some members of the aerospace industry such as the Northrop Corporation, the Boeing Company, for their use in analysing cracked structures.

The results of the above analysis have been presented in Refs. 1-5.

B. Study of Convergence of the Finite Element Method for Fracture Mechanics

A convergence proof was established for the present finite element method for solving stress intensity factors in linear fracture mechanics. The results are currently being prepared for publication. Also, a formulation for a two-dimensional finite element, based on the present displacement hybrid approach, to improve the satisfaction of stress-free conditions on a boundary segment of an element adjoining the crack surface, was completed. This element is tested for its accuracy in plane problems where the crack intersects a free surface, and the results are being prepared for publication in the form of a brief note.

C. Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement-Hybrid Finite Element Model

We have developed a finite element procedure for the calculation of modes I, II and III stress intensity factors, which vary along an arbitrarily curved three-dimensional crack front in a structural component. The present finite element model is based on a modified variational principle of potential energy with relaxed continuity requirements for displacements at the interelement boundary. The variational principle is a three-field principle, with the arbitrary interior displacements for the elements, interelement boundary displacements, and element boundary tractions as variables. The unknowns in the final algebraic system of equations, in the present model, are the nodal displacements and the three elastic stress intensity factors.

Special elements, which contain proper square root and inverse square root variations in displacements and stresses, respectively, are used in a fixed region near the crack front. Interelement displacement compatibility is satisfied, by assuming an independent interelement boundary displacement field, and using a Lagrangian multiplier technique to enforce such interelement compatibility. These Lagrangian multipliers, which are physically the boundary tractions, are assumed from an equilibrated stress field derived from three-dimensional Beltrami (or Maxwell-Morera) stress functions that are complete. However, considerable care had to be exercised in the use of these stress functions such that the stresses produced by any of these function components are not linearly dependent.

Since the method is based on a rigorous variational principle, which enforces at least on an average the conditions of interelement displacement continuity when \sqrt{r} type displacements are included in the near-tip region, the convergence of the finite element solution for nodal displacements as well as the stress intensity factors is established mathematically.

The geometry of the "basic element" used presently is a 20-node "isoparametric" brick element, with 60 degrees of freedom per element. Two options were developed: (a) a singular element for use near the crack front, wherein the stress intensity factors K_I , K_{II} and K_{III} are constant, and (b) a singular element where in the intensity factors K_I , K_{II} and K_{III} themselves vary quadratically. It is believed that this option will reduce the number of elements to be used near the crack front, in order to obtain an accurate variation of the intensity factors along the crack front.

The relevant metrics were evaluated numerically, using non-product type quadrature formulae, with proper mathematical transformations being

used when singular type functions are encountered in stresses and strains in the near-tip region.

The utility of the formulation is demonstrated through numerical results for the problem of a through-the-thickness crack in a finite width plate subjected to uniform tension. In analysing this problem, the controversy regarding the singularity where the crack tip intersects the flat surface has been recognized and attempts have been made to assess the three-dimensional influence of the flat surface on the two-dimensional intensity factors. Extensive debugging has been performed to check the numerical validity of the present procedure and related computer program.

Work is in progress to solve some realistic three-dimensional fracture problems including a slanted, semi-elliptical surface flaw in a tension plate, a quarter circular edge notch in a tension bar, and an embedded elliptical flaw in a tension bar.

The first results from the above described three-dimensional work are being published in Refs. 7 and 8.

Recently, it has been learned that Dr. Randy Peeters of AFRPL/MKPB, Rocket Propulsion Lab, Edwards Air Force Base, California is interested in using the present program developed under AFOSR grant, to solve some fracture problems in Rocket Propellant grains. Arrangements are being made by AFRPL to provide some computer time for us to be able to solve these problems.

Dr. Eric Becker and Dr. Robert Dunham of the University of Texas at Austin, who are under contract to AFRPL to do similar three-dimensional analysis, have also contacted us to seek the possibility of using our work in relation to their research. With the consent of Dr. W. J. Walker of

AFOSR, this program will be released to Dr. Peeters of AFRPL as well as to Dr. Becker, at a time when we will have satisfactorily tested our program in critical test cases of arbitrary, curved, three-dimensional cracks.

D. Elastic-Plastic Two Dimensional Fracture

We are concerned with the problem of ductile fracture under large-scale yield conditions near the crack-tip.

The theoretical formulation was completed for a finite element procedure to study the elastic-plastic response of a cracked sheet, under mixed mode conditions, when subjected to pulsating far field stresses and fully reversed loads. The magnitude of the far field stress is such that large scale yielding occurs near the crack-tip.

The first objective of this research is to compute a J (an energy-like path integral) versus δ (a representative displacement such as load point displacement or the crack opening displacement) relation for typical test specimens such as the three-point bend, center notch, and compact tension specimens, under large scale yielding conditions. By correlating with the experimental data for critical J and the corresponding data, this analysing thus aims at establishing a fracture initiation criteria for the given geometry and loading conditions.

In the formulation, the material uniaxial stress-strain curve was assumed to be of general Romberg-Osgood type. Special singular elements containing the proper singularities in stress and strain (as predicted by the works of Hutchinson, and Rice and Rosengren) have been developed. For maintaining interelement displacement compatibility, an assumed displacement hybrid finite element model has been employed.

For properly accounting for the load history (and cyclic loading) a J_2 incremental flow theory of plasticity has been employed. A vigorous variational principle for the incremental elastic-plastic analysis using the displacement hybrid model has been developed. Because of its computational efficiency, an "initial-strain" method has been adopted for the incremental elastic-plastic finite element analysis. Rigorous expressions for equilibrium check and other iterative process in the incremental analysis have been derived. An efficient solution technique for general nonlinear equations derived by the finite element method has also been developed [Ref. 6].

The material is assumed to be of arbitrary strain-hardening type. For considering the Bauschinger effect, a kinematic-hardening flow rule and subsequent yield surface representation of Prager and Ziegler has been employed. The yield condition, for simplicity, has been assumed to be of the Von Mises type. Incremental constitutive relations for both plane strain and plane stress cases have been derived in an original way for taking into account the anisotropy introduced by the kinematic hardening rule. An extension and application of this research to a different problem area, viz., indentation of a brain and calculation of fragility index for the brain tissue has been made at the request of Dr. A. S. Kobayashi of the University of Washington [Ref. 10].

The computer programs based on the above formulations are under active development, and it is anticipated that the first results would be available in two to three months [Ref. 9].

E. Elastic-Plastic Analysis of Fatigue Crack Growth

The second objective of work reported under item (d) above is to study the possible acceleration or retardation effect of crack-closure and opening

stresses on fatigue crack growth. Considerations are underway to incorporate the growth initiation criteria developed under (d) above into the study of fatigue crack propagation rate, da/dN . Since the (da/dN) is known to depend on the elastic-intensity factor range during which the crack actually remains open under cyclic loading, an accurate determination of the crack-opening and closure stresses is important. Thus, the objective of the research is to study the effects of residual plastic deformations left in the wake of the advancing crack in keeping the crack closed for a significant portion of even the tensile load cycle. This computational procedure thus clearly leads to a more reliable way of predicting the crack propagation rate, da/dN .

3. Papers Published, Based on Research Supported by AFOSR Grant

1. S. N. Atluri, A. S. Kobayashi and M. Nakagaki, "Application of an Assumed Displacement Hybrid Finite Element Procedure to Two Dimensional Problems in Fracture Mechanics," AIAA Paer No. 74-390, in Proceedings of AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974.
2. S. N. Atluri, A. S. Kobayashi and M. Nakagaki, "An Assumed Displacement Hybrid Finite Element Method for Fracture Mechanics," International Journal of Fracture Mechanics (in print), also in summary form in Proceedings of the Seventh U.S. National Congress of Applied Mechanics, Boulder, Colorado, June 1974.
3. S. N. Atluri, A. S. Kobayashi and M. Nakagaki, "Solutions for Some Two-Dimensional Fracture Problems," AIAA Journal (in print).
4. S. Atluri, A. S. Kobayashi and M. Nakagaki, "A Finite Element Program for Fracture Mechanics Analysis of Composite Materials," ASTM Special

Technical Publication (Proceedings of ASTM Symposium on Fracture Mechanics of Composites, Committee D-30 on High Modulus Fibers and their Composites, National Bureau of Standards, Gaithersburg, Maryland, Sept. 1974, in print).

5. S. Atluri, "On Hybrid Displacement and Stress Models in Solid and Fracture Mechanics," to appear in Proceedings of AICA International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, Pa., June 1975.
6. S. Atluri, "Finite Element Perturbation Analysis of Nonlinear Dynamic Response of Elastic Continua," in Proceedings of 1974 International Conference on Finite Element Methods in Engineering, University of New South Wales, Sydney, Australia, August 1974.
7. S. Atluri, K. Kathiresan and A. S. Kobayashi, "Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement Hybrid Finite Element Model," accepted for publication in Proceedings of the Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975.
8. S. Atluri and K. Kathiresan, "Analysis of an Elliptical Surface Flaw by a Three-Dimensional Finite Element Procedure," being submitted for presentation and publication at 11th Annual Meeting of the Society of Engineering Science, University of Texas at Austin, October 1975.
9. S. Atluri and M. Nakagaki, "On Computation of J- δ Relations for Large-Scale Yielding Plane Crack Problems by Finite Element Method," being submitted for presentation and publication at 11th Annual Meeting of the Society of Engineering Science, University of Texas at Austin, October 1975.

10. S. Atluri, A. S. Kobayashi and J. S. Cheng, "Brain-Tissue Fragility: A Finite Strain Analysis by a Hybrid Finite Element Method," Journal of Applied Mechanics, paper No. APMW-75-46, being presented at Applied Mechanics Western Conference, University of Hawaii, March 25, 1975.

4. Additional Papers Presented Based on Research Supported by AFOSR

1. S. Atluri, "Finite Element Analysis of Cracks in Non-Homogeneous Media," presented by invitation at NATO Advanced Study Institute on Continuum Mechanics of Geodynamics and Rock Fracture Mechanics, Reykjavik, Iceland, August 1974.
2. A. S. Kobayashi, S. Atluri and M. Nakagaki, "Finite Element Methods in Fracture Analysis of Composites," present by invitation at Symposium on Fracture of Composites, Battelle Seattle Research Center, Seattle, Wash. February 1975.
3. S. Atluri, "On Elastic-Plastic Fracture Mechanics," invited seminar at the Department of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, August 21, 1974.
4. S. Atluri, "Computational Methods in Fracture Mechanics," presented at the National Aeronautical Laboratory, Fracture Group, Bangalore, India, August 22, 1974.

Interim Report of Research on
ELASTIC-PLASTIC FINITE ELEMENT ANALYSIS OF CRACKS

February 1, 1975 - February 1, 1976

Grant AFOSR 74-2667

Satya N. Atluri
School of Engineering Science and Mechanics
Georgia Institute of Technology
Atlanta, Georgia 30332

March 1976

Interim Report on AFOSR 74-2667

Elastic-Plastic Finite Element Analysis of Cracks

1. Introduction

Reported herein is a summary of research on elastic-plastic analysis of cracks, during the period February 1, 1975 to January 31, 1976, under the support of AFOSR Grant 74-2667.

The research described has been performed at the School of Engineering Science and Mechanics, Georgia Institute of Technology, under the direction of Dr. Satya N. Atluri. Dr. Michihiko Nakagaki participated in the research as a Post-doctoral Fellow and made significant contributions. Two doctoral students, Mr. K. Kathiresan and Mr. W. H. Chen, have also been participating in the research and their doctoral theses, in the same field of research, are nearing completion.

Dr. M. E. Raville, Director, School of Engineering Science and Mechanics, has provided immense support in seeing that the research efforts were not hampered by the several institutional budgetary crises during the year.

Dr. W. J. Walker of AFOSR provided invaluable encouragement and has aided immensely in defining the practical objectives of the basic research.

The studies during this report period include: (a) the J-integral as a ductile-fracture criterion; (b) studies of finite deformation effects near the crack-tip; (c) study of stable crack growth; (d) elastic-plastic analysis of fatigue crack growth; (e) hybrid-finite element solution of fundamental three-dimensional crack problems.

A brief report on progress made on each of these efforts to date, and comments on further work are given below. A listing of publications that

have resulted from this research, papers presented at professional meetings, and seminars given at other research organizations is included.

2. Summary of Research Accomplishments

(A) The J-Integral as a Ductile-Fracture Criterion

To study the problem of ductile fracture under large-scale yielding conditions, research was conducted in formulating and developing a second generation finite-element procedure with the objectives: (i) developing circular-sector shaped embedded-singularity finite elements near the crack-tip. The correct r dependence of the dominant singular solution, corresponding to the nonlinear material model (Ramberg-Osgood law), was embedded in these near tip elements, where as the θ -dependence was approximated in each sector-element and solved for in the sense of the finite element method, (ii) maintaining continuity of displacements and tractions, between near-tip elements with singular stress/strain assumptions and the far-field elements with regular stress/strain assumptions, through a hybrid displacement finite element model, (iii) using a J_2 -flow theory of plasticity and arbitrary kinematic hardening which will accurately model the Bauschinger effect under fully reversed and cyclic loading, (iv) using an incremental finite element solution procedure that will be suitable in the limiting case of elastic-perfect-plastic materials, as well as cyclic loading situations. The "initial-stress" iteration approach was used for this purpose, (v) developing a more accurate finite element method for incremental analysis of elastic-plastic problems. The more common approach in literature is to use "constant-stress" elements, and based on stress level in each element, the whole element either yields or stays elastic. Thus in problems such as the present, where the

yield zones near the crack-tip play a dominant role in the analysis and its interpretation, in order to obtain a reasonably accurate description of the yield zone, a very fine finite element mesh is needed. However, if higher-order elements are used, and if "plasticity-correction" iterations are performed at several points within the element, it then becomes possible to give a smoother definition of the yield zone. Thus, a portion of the element, in the present formulation, can yield while the rest of the element can remain elastic.

The first opportunity to check the validity of the analysis presented itself when the writer was invited by the ASTM Task Group E-24.01 to participate in an Analysis Round Robin. The Round-Robin problem involved a three-point bend specimen, of a hypothetical material. The objectives of the analysis were (i) to check the validity of a one-parameter J-integral ductile fracture criterion, i.e., its path-independence under J_2 -flow theory, etc., and (ii) to check the empirical relations developed by Rice, Paris, Merkle and others to estimate the J-integral from single specimen test data.

The first complete solution to the above Round Robin problem was provided by the writer and his colleagues, and is published in Refs. 9 and 11. The results were also presented to the ASTM Committee E-24 at its invitation.

To test the developed analysis procedure, it has also been used in the analysis of a compact tension specimen of A533B pressure vessel steel for which rigorous experimental results have been reported by Drs. Begley and Landes at Westinghouse Research Laboratories. Excellent correlation between the present results and the reported experimental results has been noted for J-integral estimates. It has also been found that the empirical estimate of Rice, Paris and Merkle for J from single specimen test data is in significant

error in the case of a compact tension specimen.

The above results for compact tension specimen are reported in Ref. 12 and are to be presented shortly at the AIAA/ASME/SAE 17th Structures, Structural Dynamics and Materials Conference, in Valley Forge, Pennsylvania.

The above results are believed to be first conclusive analytical results to appear in literature for the analysis of realistic fracture test specimens.

Results pertaining to the analysis of general, arbitrary plane ductile fracture mechanics problems, such as cracks in plane stress sheets are being presented at the forthcoming International Conference on Piping and Pressure Vessel Technology.

The present procedure has also been used to analyze an altogether different problem, that of fragility index for brain tissue, in Ref. 8

(B) Studies of Finite Deformation Effects Near the Crack-Tip

To study the effects of crack-tip blunting, a finite-deformation, embedded singularity, elastic-plastic incremental analysis has been developed. This analysis forms a part of the doctoral dissertation of Mr. W. H. Chen. To account for both the geometric and material nonlinearities, a tangent modulus incremental method has been developed.

The developed procedure has been used to analyse a three-point bend specimen of Ni-Cr-Mo-V rotor steel, for which experimental results have been reported by the Westinghouse Research Laboratories. Excellent correlation between the present finite deformation analysis results and the cited experimental results have been noted for the critical J for fracture. The present results have been documented in a paper, to be presented at the forthcoming U.S. National Symposium on Fracture, and to be published by ASTM.

One of the most interesting conclusions of the present finite deformation analysis was that by modifying the original definition of the J-integral (as given by Rice) to account for finite deformations, a more accurate path-independence can be noticed even for paths closest to the crack-tip.

(C) Study of Stable Crack Growth

To study the phenomenon of slow stable crack growth, research has been initiated into (i) devising an efficient scheme to translate the entire set of sector-shaped singular elements in the direction of stable growth. Thus, at successive instants of time, a 'new' crack-tip is generated, with the attendant 'new' singular states of stress and strain, (ii) since the J-integral is invalid when there is unloading as in the case of stable growth, an alternate criteria based on global energy balance is being developed.

The first results from this analysis are expected in the two-to-three months, and will form the second part of Mr. W. H. Chen's thesis.

(D) Elastic-Plastic Analysis of Fatigue Crack Growth

Using the above described idea of moving the "singularity", research has been initiated into the simulation of fatigue-crack-growth. The objective is to study the acceleration and retardation effects of crack-closure on fatigue crack growth rate (da/dN). Since (da/dN) is known to depend on the elastic-intensity-factor range during which the crack actually remains open under cyclic loading, an accurate determination of crack-opening and closure stresses is important. This crack closure is due to the effect of residual plastic deformations left in the wake of the advancing crack.

Since, in the present study, fatigue crack growth is simulated by moving the entire set of "singular elements" in the direction of growth, rather

than just moving the crack-tip to a neighboring node, the need for using a very fine mesh near the crack-tip is obviated. The present procedure thus clearly leads to a more efficient and reliable way of predicting (da/dN) . It is believed that this aspect of research will result in a significant contribution to the literature on estimating (da/dN) .

(E) Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement Hybrid Finite Element Model

During the present reporting period, the formulation and development has been completed for a "three-dimensional singular element" for the calculation of modes I, II and III stress intensity factors that vary along an arbitrarily curved three-dimensional crack front. The formulation procedure for this "hybrid, 3-D cracked element" has been described in the previous interim report.

The development of this 3-D singular element proved to be a very ambitious task, with challenging numerical problems, of ill-conditioning of matrices, round-off errors, and numerical integration of functions involving singularities. Much of the time was spent in solving these associated numerical problems, which appeared in the beginning to be routine.

The successful formulation of the 3-D singular element has been presented and published at the 3rd International Conference on Structural Mechanics in Reactor Technology, held at the University of London, in Sept. 1975.

To test the developed 3-D singular element, the need arose for a general purpose 3-D finite element program into which the present singular element can be incorporated. Much to the regret of the writer, such a program was

not available through open sources. It was also felt that the development of an efficient, general purpose 3-D computer program, involving primarily computer software problems, did not fit in the frame work of basic research to be supported by AFOSR. Instead it was believed that, since the purpose of the present basic research is to develop 3-D cracked elements, it would suffice to test the element through a crude ("brute force") 3-D finite element program. The credit for developing this crude 3-D program goes to Mr. Kathiresan. This program is necessarily inefficient (in terms of running time) since no expertise on computer software such as dynamic storage, and overlay programming was employed.

Extensive debugging, to check the present procedure and its incorporation into the above described 3-D finite element program, has been performed and is still underway. The first problem that was solved was that of a through-the-thickness crack in a finite width plate subjected to uniform remote tension. In analysing this problem, the controversy regarding the singularity where the crack-tip intersects the flat surface has been recognized and alternative attempts have been made to assess the three-dimensional influence of the flat surface on the two-dimensional intensity factors. These results have been presented at the 12th Annual Society of Engineering Science Meeting at Austin, Texas in October 1975.

Work is in progress to solve some practical three-dimensional fracture problems including embedded as well as surface flaws of circular or elliptical geometry in tension bars, and surface flaws in pressurized cylinders. Some of these results are to be presented by invitation at the forthcoming International Conference on Pressure Vessel Technology.

The preliminary version of the 3-D cracked element has been supplied, with the approval and encouragement of Dr. W. J. Walker to (i) the Northrop Corporation, (ii) Dr. J. H. Gallagher of Wright-Patterson Air Force Base, and (iii) Dr. Randy Peeters of AFRPL and Dr. Collingwood of Thiokol.

The present work on the 3-D cracked element development is also being presented at a "Workshop on 3-D Fracture Analysis" at Battelle Columbus Laboratories, Columbus, Ohio on April 26-28, 1976.

Concluding Remarks and Acknowledgements

This research effort, as perhaps in any other, has had its frustrating moments. The writer wishes to take this opportunity to express his deep sense of appreciation to his associates in research, especially Dr. Nakagaki, Mr. Kathiresan and Mr. Chen for withstanding the pressure to meet the objectives of this research in a timely fashion, and to stay abreast. He also appreciates their dedication and their sharing of the feeling of excitement of coming years of research.

3. Papers Published Based on Research Supported by AFOSR

1. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "Application of an Assumed Displacement Hybrid Finite Element Procedure to Two Dimensional Problems of Fracture Mechanics," AIAA paper No. 74-390 in Proceedings of AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conf., Las Vegas, Nevada, April 1974.
2. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "An Assumed Displacement Hybrid Finite Element Model for Linear Fracture Mechanics," International Journal of Fracture, Vol. 11, No. 2, pp. 257-271, April 1975; Also a summary appears in Proceedings of 7th U.S. National Congress of Applied Mechanics, ASME, March 1975, p. 104.

3. Atluri, S. N., Kobayashi, A. S., and Nakagaki, M., "Fracture Mechanics Application of an Assumed Displacement Hybrid Finite Element Procedure," AIAA Journal, Vol. 13, No. 6, pp. 734-740, June 1975.
4. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "A Finite Element Program for Fracture Mechanics Analysis of Composite Materials," Fracture of Composites, ASTM Special Technical Publication, 593, American Society for Testing and Materials, pp. 86-98, November 1975.
5. S. N. Atluri, "On Hybrid Finite Element Models in Solid Mechanics," Advances in Computer Methods for Partial Differential Equations, (R. Vishnevetsky, Editor) Rutgers University Press, pp. 351-362, June 1975.
6. S. N. Atluri, "Finite-Element Perturbation Analysis of Nonlinear Dynamic Response of Elastic Continua," Finite Element Methods in Engineering, Proceedings of 1974 International Conference on Finite Element Methods in Engineering (Pulmano and Kabaila, Editors), Unisearch, Ltd., Sydney, Australia, pp. 521-538, September 1974.
7. S. N. Atluri, K. Kathiresan, and A. S. Kobayashi, "Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement Hybrid Finite Element Procedure," Paper No. L-7/3, Transactions of the Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975.
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4. Papers Presented at Professional Meetings

16. AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974, paper title same as cited in Ref. 1.
17. 7th U.S. National Congress of Applied Mechanics, Boulder, Colorado, June 1974, paper title same as cited in Ref. 2.
18. 1974 International Conference on Finite Element Methods in Engineering, University of New South Wales, Sydney, Australia, Aug. 29-31, 1974, paper title same as cited in Ref. 6.
19. NATO Advanced Study Institute on Continuum Mechanics Aspects of Rock Fracture and Geodynamics, Reykjavik, Iceland, Aug. 1974, presented and invited paper, "Finite Element Analysis of Cracks Between Dissimilar Media."
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21. Conference on Fundamental Aspects of Deformation and Fracture of Composite Materials, Battelle Seattle Research Center, Seattle, Washington, Feb. 1975, presented an invited paper, "Stress Intensity Factors of Cracked Orthotropic Plates," (presented by A. S. Kobayashi).
22. ASME Joint Western and National Applied Mechanics Conference, University of Hawaii, Honolulu, March 1975, paper title same as cited in Ref. 8. (presented by A. S. Kobayashi).

23. AICA International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, Pa. June 1975, paper the same title as cited in Ref. 5.
24. AFRPL/Edwards AFB Contractors Meeting, California Institute of Technology, Pasadena, Calif. May 1975, invited presentation on, "3-D Cracked Elements,"
25. ASTM Committee E-24 Meeting, 9th U.S. National Symposium on Fracture Mechanics, Pittsburgh, Pa., August 1975, invited presentation, "Analysis of Large-Scale Yielding Fracture Problems."
26. Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975, paper title same as cited in Ref. 7.
- 27-28. 10th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, October 1975, paper titles same as cited in Ref. 9 and 10.
29. "Workshop on 3-D Fracture Analysis," Battelle Columbus Laboratories, Columbus, Ohio, April 1976, invited presentation on "3-D Hybrid Cracked Elements."
- 30-34. Five other papers are to be presented, with titles cited as Refs. 11, 12, 13, 14, and 15.

5. Seminars

1. "On Elastic-Plastic Fracture Mechanics," at Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, Aug. 21, 1974.
2. "Computational Methods in Fracture Mechanics," National Aeronautical Lab, Bangalore, India, Aug. 22, 1974.
3. "Computational Fracture Mechanics," College of Engineering, Boston Univ. Boston, Mass., January 1976.

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Research on
ELASTIC-PLASTIC FINITE ELEMENT ANALYSIS OF CRACKS

February 1, 1976-February 1, 1977
Grant AFOSR 74-2667

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Elastic-Plastic Finite Element Analysis of Cracks

1. Introduction

Reported herein is a summary of research on elastic-plastic analysis of cracks, during the period February 1, 1976 to January 31, 1977, under the support of AFOSR Grant 74-2667.

The research described has been performed at the School of Engineering Science and Mechanics, Georgia Institute of Technology, under the direction of Dr. Satya N. Atluri. Dr. Michihiko Nakagaki participated in the research as a Post-doctoral Fellow and made significant contributions. Two doctoral students, Mr. K. Kathiresan and Mr. W.H. Chen, have also been participating in the research as part of their Doctoral dissertations; Dr. Kathiresan obtained his Ph.D. in December 1976; and Mr. Chen is expected to graduate in July 1977.

Dr. M. E. Ravioli, Director, School of Engineering Science and Mechanics, has provided immense support in seeing that the research efforts were not hampered by the several institutional budgetary crises during the year.

Dr. W. J. Walker of AFOSR provided invaluable encouragement and has aided immensely in defining the practical objectives of the basic research.

The studies during this report period include: (a) crack growth initiation and its stability in plane problems of ductile fracture and (b) hybrid finite element methods for three-dimensional elastic and elastic-plastic fracture analysis.

A brief report on progress made on each of these efforts to date, and comments on work in progress are given below. A listing of publications that have resulted from this research, papers presented at professional meetings, and Seminars given at other research organizations is included.

2. Summary of Research Accomplishments

A considerable portion of the studies in the present grant period was aimed at understanding the problem of crack growth initiation and its stability in plane problems of ductile fracture, in the presence of large-scale yielding near the crack-tip. To this end, attention has been focused on such concepts as the J-integral, the crack-tip opening displacement, the R-curve method, and nonlinear energy release rate.

A finite element methodology, based on the hybrid finite element technique and employing an incremental flow theory of plasticity, was developed to study in-service ductile fracture situations. This procedure is anticipated to be capable of analyzing not only the conditions for crack-growth initiation, but also the stability of such crack growth. This procedure is capable of treating plane ductile fracture problems under large-scale yielding conditions, and when the material properties can be characterized as elastic-plastic with arbitrary strain-hardening. Specific achievements of the present research are:

(1) Circular-sector shaped "embedded-singularity" elements were developed for use as finite elements near the crack-tip. The correct dominant singular behavior for strains and stresses, in the radial coordinate r , corresponding to the well known Rice-Rosengren-Hutchinson asymptotic solution for power law hardening materials, was built into these elements. The θ -dependence of the asymptotic solution was, however, approximated in each sector by a quadratic polynomial. Continuity of displacements and tractions between these near-tip "singular" elements and the far-field "regular" elements was enforced through a hybrid-displacement finite element procedure, analogous to the procedure adopted by the author and his colleagues in linear-elastic fracture problems [2]. The incremental plastic flow was described by a Prager-Ziegler kinematic hardening law, which accounts for the Bauschinger effect during unloading and thus

situations of stable crack growth and fatigue crack growth can be treated easily. To start with, crack-tip blunting and other finite geometry changes near the crack-tip were ignored, and thus the analysis was restricted to a small-deformation assumption. This procedure was used to solve a hypothetical 3-point bend test specimen which was suggested, as a standard test for analytical solutions, by the ASTM Committee E-24 Round Robin Group on Elasto-Plastic Fracture Criteria. A detailed analysis of these results, with various fracture criteria in mind, was presented in (Atluri and Nakagaki [9], and Atluri and Nakagaki [12]). However, no other independent solution, from members of the Round Robin, with which to compare our results was available.

(2) The above small-deformation analysis procedure was used to analyze a compact tension cracked specimen, of A533B steel, for which experimental fracture data was reported by the Westinghouse Research Laboratories. The thickness of the experimental specimen was roughly $1/2$ - $1/3$ of the characteristic in-plane dimensions. Thus it was not a priori certain whether the conditions near the crack-tip can be characterized mathematically as those of plane stress or of plane strain. Thus the problem was analyzed using two types of two-dimensional approximations. Also, the experimental uniaxial stress/strain curve supplied to us by the Westinghouse Laboratories was seen to possess a yield point instability, which the numerical procedure could not account for. Thus, two different, smooth, Ramberg-Osgood type curves were used to fit the actual experimental stress-strain curve. From these analyses, it was found that:

- (i) the mathematical characterization of the material-property data had no significant effect on the results,
- (ii) the empirical formula of Rice et al. for estimating J was in significant error in the case of the compact tension specimen, and
- (iii) the computed plane-stress values for J were much closer to the

experimental data, as compared to the plane-strain values which differed by a factor of about 3. Thus it was concluded that the plane-stress conditions more accurately characterize the plastic flow near the crack-tip in "small" specimens. These results are documented in detail in (Atluri and Nakagaki [12]).

However, even the computed plane-stress values for J, based on the present analysis with a "non-growing stationary crack" assumption, differed by about 15-20% from the cited experimental results for J at fracture. From subsequent private communications with Begley and Landes, who conducted the experiments, it was learned that, in fact, there may have been stable crack growth, which was not monitored, in the experiments. This problem is currently being resolved, through the finite element modeling of a growing crack, and the understanding of global energy balances during sustained stress, stable crack growth.

(3) To delineate the effects of crack-tip blunting, and the attendant finite geometry changes near the crack-tip, under large-scale plastic yielding conditions, a finite-deformation, embedded singularity, elastic-plastic incremental finite element procedure was developed [13]. The test problem was that of a 3-point bend fracture test specimen, the experimental data for which was reported and made available to us by Westinghouse. A detailed comparison of the results, for stresses, strains, displacements, and J, from this finite deformation analysis, and those obtained using the above small-deformation analysis was made. Significant conclusions of the above comparison were: (i) notwithstanding the crack-tip blunting, J still remains as a valid parameter to characterize the severity of the near-tip condition, (ii) since the well-known Rice's definition of J is invalid for finite-deformation, a modified definition must be derived. This new definition involves the strain-energy density function that is dependent on Piola-Lagrange (unsymmetric) stress tensor, and the trac-

tion term appearing in the J-integral should be interpreted as Piola-Lagrange tractions in the deformed geometry as referred to the undeformed configuration. When this modified definition for J is employed, a more accurate path-dependence (within $\pm 1.5\%$) was noted for J, as compared to the small-deformation case wherein the J value on paths closest to the crack-tip was about 15% lower than that on paths in the far-field. A detailed discussion of these results was presented in (Atluri, Nakagaki, and Chen [13]).

One interesting observation was that all load-point displacement levels, including that at fracture, there was an excellent correlation (within $\pm 4\%$) between the presently computed results for J and those in the Westinghouse experiments. This suggested that no appreciable stable crack growth may have been present in the experiment. Subsequent private communications with Dr. Landes at Westinghouse appeared to confirm this. Thus because of the particular specimen geometry and loading conditions, 3-point bend specimen may offer the advantage of precisely measuring J_{IC} for crack growth initiation for a given ductile material.

(4) Further refinements, such as the incorporation of a "knee-correction", to achieve better convergence of plastically adjusted stresses, and a more accurate incremental elastic-plastic law, were made in the above finite deformation analysis. The details of the nature and form of singularity of actually computed strains and stresses, and their variation in the angular coordinate near the crack-tip were critically reviewed and compared with the small-deformation analysis, using J_2 -deformation plasticity theory, of Hutchinson, and Rice and Rosengren. These results are discussed in detail in (Atluri and Nakagaki [17]).

It was found that [17] in the above finite element analysis, that strain-singularities still existed at the root of the blunted notch. But,

because of the finite root radius of the blunted notch, there can only be a strain concentration, however large, at the tip of the notch. The effect of this strain concentration, as opposed to the strain-singularity at the blunted crack tip, on the fracture criteria is not understood. Thus, alternate ways to incorporate a strain-concentration, instead of a strain-singularity in finite-elements near the blunted crack-tip, are currently being studied.

(5) The refined finite-deformation analysis procedure was used to analyze two fracture test specimens, of 3-point bend and compact tension types, of configurations identical to those in the experiments of Westinghouse Laboratories. The obtained numerical data was analyzed with both the J and COD concepts in mind, and the following conclusions were drawn: (i) the computed J correlated well with the cited experimental results for both the specimens; (ii) J was found to be directly correlated with COD for these test specimens. Both the above sets of data indicated that, for these specimens, $J = 1.44\sigma_y$ (COD) for moderately hardening materials, where σ_y is the yield stress. Thus it can be seen that COD may also be used as a fracture criterion; (iii) however, it was found that the computation of COD, as essentially a near-tip geometric quantity, for arbitrary plane problems of ductile fracture, is not well defined. Attempts to correlate COD with such near-tip geometrical quantities such as the crack-opening at points where the elastic-plastic boundary intersects the crack-profile were found to be discouraging; (iv) the empirical formula of Rice et al. for J was once again found to be inaccurate for the compact tension specimens. These results are elaborated upon in (Atluri, Nakagaki, and Chen [15]).

Once again, the computed J for the compact tension specimen, based on the finite element modeling of a non-growing crack, were found to be about 15-20% less than in the experiments. Again this suggested that there may, in fact, have been a subcritical crack growth prior to fracturing in the experiments.

Thus there is further motivation to continue our present studies on stable crack growth.

(6) Finite Element Modeling of Finite Strain Concentration Near the

Blunted-Tip: The need for developing near-tip finite elements which allow for a representation of large, but finite strain-concentration near the root of the blunted notch has been discussed in the continuation proposal for the present AFOSR grant, dated November 1976. This task has now been completed and is to be presented shortly at the forthcoming 4th International Conference on Fracture at Waterloo, Canada, June 1977.

(7) Study of Stable Crack Growth: To study the phenomenon of slow stable crack growth, research has been directed at devising an efficient scheme to translate the entire set of sector-shaped singular elements in the direction of stable growth, thus simulating crack growth. This theoretical development has been successfully implemented in the computer program. This finite-element modeling of crack growth has been the most interesting and ambitious numerical development so far in the present grant effort. Preliminary results concerning global energy balances during crack growth, which represent the first such results so far in the literature, are currently being prepared for publication. It is planned to present these and other results anticipated in the coming months, at the forthcoming ASTM Conference on Ductile Fracture in October 1977, and at the ASME Winter Annual Meeting, November 1977.

(8) Elastic-Plastic Analysis of Fatigue Crack-Growth: Using the above described idea of "moving the singularity", work is underway in the simulation of fatigue crack growth. The objective is to study the acceleration and retardation effects of crack-closure on the fatigue crack growth rate, (da/dN) . Since (da/dN) is known to depend on the elastic-intensity factor range during which the crack actually remains open under cycle loading, an

accurate determination of crack-opening and closure stresses is important. Since, in the present study, fatigue crack growth is simulated by moving the entire set of "singular elements" in the direction of crack growth, rather than just moving the crack-tip to a neighboring node, the need for using a very fine mesh near the crack-tip is obviated. The present procedure thus clearly leads to a more efficient and reliable way of predicting (da/dN) ; and it is believed that this aspect of research will result in a significant contribution to the literature on estimating (da/dN) .

(9) Three Dimensional Linear Fracture Mechanics Analysis Using a Crack Element Based on Hybrid-Displacement Model:

In the previous months, the development of a hybrid-displacement finite element method for 3-dimensional linear elastic fracture analysis has been completed. The procedure has been exhaustively tested in analyzing various basic problems of 3-dimensional fracture that are of current interest. These include: semi-elliptical surface flaws of various aspect ratios and various crack-depth ratios in plates under tension and bending; semi-elliptical surface flaws of various aspect ratios and various depth ratios on the inner and outer surface of thick shells; and semi-elliptical surface and quarter-elliptical corner cracks near fastener holes in plates, typical of aircraft structural configurations. Moreover, the method was completely checked as to its accuracy and convergence in several other problems such as Sneddon's embedded penny shaped crack, buried elliptical cracks, and several other problems, for which analytical solutions exist. Also, the question of the optimum size of a singular element near the crack front has been studied thoroughly. All these results, along with the detailed mathematical derivations, have been presented in a recent Ph.D. thesis by Kathiresan [18], and are submitted to AFOSR for approval for publication as an AFOSR Technical Report [19]. The results for various problems have also been published

in open literature [7,10,14,16,18,23,24,25].

The preliminary version of the 3-D cracked element has been supplied, with the approval and encouragement of Dr. W. J. Walker to (i) the Northrop Corporation, (ii) Dr. J. H. Gallagher of Wright-Patterson Air Force Base, and (iii) Dr. Randy Peeters of AFRPL and Dr. Collingwood of Thiokol.

The above formulation for a hybrid 3-D crack element for homogeneous materials, is being extended to problems of cracks at bi-material interfaces, under support of the Edwards Air Force Base, California.

Various inquiries concerning the above 3-D crack element continue to be received and it is planned that any release of the computer coding of the element will be coordinated with Dr. Walker, Program Manager, at AFOSR.

The present work on 3-D element has been presented at a "Workshop on 3-D Fracture Analysis" at Battelle Columbus Labs, Columbus, Ohio, on April 26-28, 1976, and at various other conferences as listed hereunder.

Concluding Remarks and Acknowledgements: The writer wishes to take this opportunity to express his deep sense of appreciation to his colleagues, Drs. Nakagaki and Kathiresan, and Mr. Chen for their dedicated efforts in making this past year the most fulfilling in terms of new research insights gained on the fracture behavior of solids. He also sincerely thanks Dr. W. J. Walker who provided immense encouragement and also insights into the real world problems of fracture in Aerospace Structures.

PAPERS PUBLISHED BASED ON RESEARCH SUPPORTED BY AFOSR

1. S.N. Atluri, A.S. Kobayashi, and M. Nakagaki, "Application of an Assumed Displacement Hybrid Finite Element Procedure to Two Dimensional Problems of fracture Mechanics", AIAA paper No. 74-390 in Proceedings of AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974.

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 13. S.N. Atluri, M. Nakagaki, and W.H. Chen, "Fracture Analysis Under Large-Scale Yielding: A Finite-Deformation, Embedded Singularity, Elasto-Plastic Incremental Finite Element Solution", accepted for publication in STP601, ASTM Special Technical Publications, to appear 1977.
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 16. S.N. Atluri, and K. Kathiresan, "On A 3-D Singularity Element for Computation of Mixed-Mode Stress Intensities" in NASA CP-2001, pp. 267-275 (Nov. 1976), Proceedings of 13th Annual Meeting of Society of Engineering Science and also in abstract form in "Three-Dimensional Fracture Analysis", Ed. by L.E. Hilbert, Proceedings of a Workshop held at Battelle Columbus Labs, April 1976), pp. 11-14.
 17. S.N. Atluri, and M. Nakagaki, "Stress Analysis of Cracks in Elasto-Plastic Range" to appear in Proceedings of 4th International Conference on Fracture, ASME, University of Waterloo, Ontario, Canada, June 1977.
 18. K. Kathiresan, "Three Dimensional Linear Elastic Fracture Analysis by a Displacement Hybrid Finite Element Model", Ph.D. Thesis, School of

ESM, Georgia Institute of Technology, September 1976.

19. S.N. Atluri, and K. Kathiresan, "Hybrid Crack Element for 3-D Fracture Analysis" submitted to AFOSR for approval as AFOSR-TR, 1977.
20. S.N. Atluri, and M. Nakagaki, "J-Integral Estimation Procedures" accepted for publication in AIAA Journal, to appear, 1977.
21. W.H. Chen, "Finite Deformation, Elasto-Plastic Analysis of Cracks", Ph.D. Thesis, Georgia Institute of Technology, to appear, June 1977.
22. J.S. Cheng, S.N. Atluri, and A.S. Kobayashi, "Hybrid Displacement Finite Element Method For Nonlinear Elastic and Elastic-Plastic Analyses", Proceedings of 18th AIAA /ASME/SAE Structures, Structural Dynamics and Materials Meeting, San Diego, Calif. March 1977, pp. 279-290.
23. S.N. Atluri, and K. Kathiresan, "Outer and Inner Surface Cracks in Thick Walled Pressure Vessels", to appear in Transactions of 4th International Conference on Structural Mechanics in Reactor Technology, San Francisco, Calif., August 1977.
24. S.N. Atluri, M. Nakagaki, K. Kathiresan, and H.C. Rhee, "Fracture Analysis of Structures Under Combined Mode Loading", Invited paper, to appear in Proceedings of 2nd EMD Specialty Conference, ASCE, University of North Carolina, Raleigh, NC, May 1977.
25. S.N. Atluri, and K. Kathiresan, "Cracks Near Fastener Holes and Surface Flaws in Bending", being submitted to AIAA Journal.
26. M. Nakagaki, W.H. Chen, and S.N. Atluri, "Finite Element Modeling of Stable Crack Growth", being submitted for presentation and publication at ASTM Special Symposium on Elastic-Plastic Fracture, Nov. 1977.
27. M. Nakagaki, W.H. Chen, and S.N. Atluri, "Global Energy Balance Relations During Sustained Stress Crack Growth", being submitted for presentation and publication at ASME Winter Annual Meeting, Applied Mechanics Division, Dec. 1977.

PAPERS PRESENTED AT PROFESSIONAL MEETINGS

28. AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974, paper title same as cited in Ref. 1.
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30. 1974 International Conference on Finite Element Methods in Engineering, University of New South Wales, Sydney, Australia, Aug. 29-31, 1974, paper

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35. AICA International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, PA, June 1975, paper the same title as cited in Ref. 5.
36. AFRPL/Edwards AFB Contractors Meeting, California Institute of Technology, Pasadena, Calif., May 1975, invited presentation on "3-D Cracked Elements".
37. ASTM Committee E-24 Meeting, 9th U.S. National Symposium on Fracture Mechanics, Pittsburgh, PA, August 1975, invited presentation, "Analysis of Large-Scale Yielding Fracture Problems".
38. Third International Conference on Structural Mechanics in Reactor Technology, University of London, Sept. 1975, paper title same as cited in Ref. 7.
- 39-40. 10th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, October 1975, paper titles same as cited in Ref. 9 and 10.
41. "Workshop on 3-D Fracture Analysis", Battelle Columbus Lab., Columbus, Ohio, April 1976, invited presentation on "3-D Hybrid Cracked Elements".
42. 8th South Eastern Conference on Theoretical and Applied Mechanics, VPI and SU, Blacksburg, VA, April 1976, paper title same as cited in Ref. 11.
43. 17th AIAA/ASME SDM Conference, Valley Forge, PA., May 1976, paper title same as cited in Ref. 12.
44. 10th U.S. National Symposium on Fracture, Philadelphia, August 1976, title same as cited in Ref. 13.

- 45. 13th Annual Meeting of Society of Engineering Science, NASA-Langley, Hampton, VA., Nov. 1976.
- 46. 18th AIAA/ASME/SAE SDM Conference, San Diego, Calif., March 1977, title same as cited in Ref. 22, (presented by J.S. Cheng).
- 47-50. Papers to be presented, as cited in Refs. 23, 24, 26, and 27 respectively.

SEMINARS

- 1. "On Elastic-Plastic Fracture Mechanics", at Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, Aug. 21, 1974.
- 2. "Computational Methods in Fracture Mechanics", National Aeronautical Lab, Bangalore, India, Aug. 22, 1974.
- 3. "Computational Fracture Mechanics", College of Engineering, Boston Univ., Boston, Mass., January 1976.

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

OFFICE OF
THE DIRECTOR OF
FINANCIAL AFFAIRS

April 5, 1977

Ms. Joan Marshall
AFOSR/PM
Building 410
Bolling AFB, D. C. 20332

Dear Ms. Marshall:

Enclosed in duplicate is the Grant Fiscal Report for grant number AFOSR-74-2667 covering the period February 1, 1974 through January 31, 1977.

If you have questions or desire additional information, please let us know.

Sincerely yours,


Evan Crosby
Associate Director of
Financial Affairs

EC/bs
enclosures:
cc: Dr. S. N. Atluri
Mr. E. E. Renfro ✓
Mr. A. H. Becker
File E-23-608

E-23-608

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH GRANT FISCAL REPORT

Submit in duplicate. Interim reports are required each year for grants of more than one year's duration. Submit final report as soon as possible but not more than 90 days after the end of the grant period.

FROM: Georgia Institute of Technology Atlanta, Georgia 30332		TO: AFOSR/PM Bldg 410 Bolling AFB, D.C. 20332	REPORT DATE 3-22-77	GRANT NO. AFOSR- 74-2667
TYPE OF REPORT		REPORT PERIOD		
<input type="checkbox"/> FINAL <input checked="" type="checkbox"/> INTERIM		FROM: INCEPTION TO: January 31, 1977		
TOTAL PROJECT COST NEGOTIATED \$ 147,391.	TOTAL GRANT AMOUNT \$ 115,391.	AGREED PERCENT OF TOTAL PROJECT COST REPRESENTED BY GRANT AMOUNT 74 %		
DIRECT COSTS		GRANT FUNDS EXPENDED		
SALARIES AND WAGES				
PRINCIPAL INVESTIGATOR		25,678.72		
SENIOR SCIENTIFIC		-		
OTHER		39,192.21		
PERMANENT EQUIPMENT				
EXPENDABLE SUPPLIES AND MATERIALS		953.04		
TRAVEL		3,217.00		
PUBLICATION COSTS		215.60		
COMPUTER COSTS		515.56		
EMPLOYEE BENEFITS		2,517.93		
OTHER (Specify)				
TOTAL DIRECT COSTS		72,290.06		
INDIRECT COSTS: 68 % OF S & W		42,951.20		
TOTAL GRANT FUNDS EXPENDED		115,241.26		
I CERTIFY THAT THE PRINCIPAL INVESTIGATOR(S) HAS (HAVE) DEVOTED APPROXIMATELY % OF HIS (THEIR) TIME TO PERFORMANCE OF THE RESEARCH UNDER THE GRANT. I FURTHER CERTIFY THAT THIS FISCAL REPORT IS CORRECT; THAT ALL EXPENDITURES REPORTED WERE FOR APPROPRIATE PURPOSES; AND THAT THE GRANTEE HAS EXPENDED FROM NON-FEDERAL FUNDS IN THE COST CATEGORIES LISTED ABOVE, IN PERFORMANCE OF THE RESEARCH UNDER THIS GRANT, THE AMOUNT OF		32,174.73		
TOTAL ACTUAL PROJECT COST FOR REPORT PERIOD		147,415.99		
ON FINAL FISCAL REPORT ONLY: IF ACTUAL TOTAL PROJECT COST IS LESS THAN NEGOTIATED TOTAL PROJECT COST, MULTIPLY THE ACTUAL TOTAL PROJECT COST BY THE AGREED PERCENTAGE REPRESENTED BY THE GRANT AMOUNT AND ENTER RESULT HERE. (NOTE: Regardless of the above computation, any unexpended or uncommitted grant funds must be returned to AFOSR in accordance with the paragraph of the Grants Brochure "Unexpended Funds and Earned Interest")				
DEDUCT THIS AMOUNT FROM TOTAL GRANT AMOUNT				
ADD INTEREST EARNED				
REFUND DUE (return this amount with the final report by check made payable to the Treasurer of the United States)				
<div style="text-align: center;">  AFOSR ECST, AFOSR, BOLLING AFB </div>				
SIGNED	TITLE			
S. Atluri	Professor			
Evan Crosby	Associate Director of Financial Affairs			
If the actual cost in this category varies more than 10% from the estimate, provide an explanation of the variance on the reverse of this form. Firm outstanding commitments are considered as costs.				

Final Scientific Report
of
Research on
"Finite Element Elastic-Plastic Analysis of Cracks"

Supported by
AFOSR Grant 74-2667
February 1, 1974 - February 1, 1978
(Dr. William J. Walker, Program Manager)

Submitted by

Satya N. Atluri
School of Engineering Science and Mechanics
Georgia Institute of Technology
Atlanta, Georgia 30332

March 1978

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Atlanta, Georgia 30332

March 1978

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR-	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Finite Element Elastic-Plastic Analysis of Cracks		5. TYPE OF REPORT & PERIOD COVERED Final Scientific 2/1/74 - 2/1/78
		6. PERFORMING ORG. REPORT NUMBER GIT-ESM-78-2
7. AUTHOR(s) Satya N. Atluri		8. CONTRACT OR GRANT NUMBER(s) AFOSR 74-2667
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Engineering Science and Mechanics Georgia Institute of Technology Atlanta, GA 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 681307 9782-04 161102 F1
11. CONTROLLING OFFICE NAME AND ADDRESS AFOSR/NA (W. J. Walker, Program Manager) Building 410 Bolling AFB, Washington, D. C. 20332		12. REPORT DATE March 1978
		13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release, Distribution Unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) Approved for Public Release, Distribution Unlimited.		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) (Cont. on Reverse) 2-D Hybrid Crack Element; Curved Cracks; Anisotropic Materials, free-surface effects; Convergence of Hybrid Element Crack Solutions; 3-D Hybrid Crack Element; Direct Solution for K_I , K_{II} and K_{III} ; semi-elliptical surface flaws in plates; Tension and Bending; Semi-elliptical Surface Flawed Cylinders;		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (Cont. on Reverse) This is a summary of research performed in the areas: (a) development of two-dimensional hybrid finite element procedures to calculate stress intensity factors corresponding to general $r^{-\alpha}$ stress singularities in isotropic as well as anisotropic materials; (b) study of the convergence of the assumed displacement hybrid finite element procedure in fracture mechanics problems;		

19. Corner Cracked Fastener Holes; Cracks in Adhesively Bonded Metallic Laminates; Ductile Fracture; Large-Scale Plastic yielding; Plastic Hybrid Crack Element; fracture Initiation; J-Integral; Finite Deformation Effects; Crack-tip Blunting; Compact-Tension Specimen; 3-pt Bend Specimen; Center-Crack Specimen; Plane Stress vs. Plane Strain; Finite Deformation J; COD; CTOA; Stable Crack Growth; Elastic-Plastic Energy Release Rate to Crack-tip; Process Zone Energy Release Rate; Translation of Singularities, Stability Criteria, Crack Closure; Fatigue Crack Growth.
20. (c) developement of a three-dimensional hybrid finite element procedure to calculate the elastic combined mode stress-intensity factors K_I , K_{II} and K_{III} that vary along an arbitrarily curved three-dimensional crack front; (d) hybrid finite element solutions of fundamental three-dimensional crack problems; (e) development of a two-dimensional finite element procedure for analyzing plane problems of fracture involving large-scale yielding conditions and under cyclic loading; (f) study of the J-integral as a ductile fracture initiation condition; (g) studies of finite deformation effects near the crack-tip; (h) analysis of stable crack growth under rising load and study of criteria for loss of stability of growth in ductile materials; and (i) elastic-plastic analysis of effects of crack closure on fatigue crack growth rates.

1. INTRODUCTION

Reported herein is a summary of research on elastic-plastic analysis of cracks, during the period February 1, 1974 to February 1, 1978, under the support of AFOSR Grant 74-2667.

The research described has been performed at the School of Engineering Science and Mechanics, Georgia Institute of Technology, under the direction of Satya N. Atluri. Three doctoral theses were written, by Drs. Nakagaki, Kathiresan, and Chen, based on research performed under this grant; these are referenced at the end of this report. Dr. Nakagaki later participated in this research as a Post-Doctoral Fellow and made significant contributions.

Dr. W. J. Walker of AFOSR provided invaluable encouragement and has aided immensely in defining the practical objectives of the basic research.

2. AREAS OF RESEARCH

The studies under the present grant included: (a) development of two-dimensional hybrid finite element procedures to calculate stress intensity factors corresponding to general $r^{-\alpha}$ stress singularities in isotropic as well as anisotropic materials; (b) study of the convergence of the assumed displacement hybrid finite element procedure in fracture mechanics problems; (c) development of a three-dimensional hybrid finite element procedure to calculate the elastic combined mode stress-intensity factors K_I , K_{II} , and K_{III} that vary along an arbitrarily curved three-dimensional crack front; (d) hybrid finite element solutions of fundamental three-dimensional crack problems; (e) development of a two-dimensional finite element procedure for analyzing plane problems of fracture involving large-scale yielding conditions and under cyclic loading; (f) study of the J-integral as a ductile fracture initiation condition; (g) studies of finite

deformation effects near the crack-tip; (h) analysis of stable crack growth under rising load and study of criteria for loss of stability of growth in ductile materials; and (i) elastic-plastic analysis of effects of crack closure on fatigue crack growth rates.

A brief report on the salient results obtained in each of the above studies; a listing of publications that have resulted from this research; papers presented at professional conferences; and seminars given at other organizations, are given in the following.

3. SUMMARY OF RESEARCH ACCOMPLISHMENTS

A. Two-Dimensional Linear Elastic Fracture Analysis.

A basic procedure, based on an assumed displacement hybrid finite element method, to calculate the stress intensity factors for mixed mode behaviour of arbitrary shaped curved cracks in plane stress and plane strain problems involving anisotropic, nonhomogeneous, but linearly elastic materials has been developed [Refs. 1, 4]. The attendant finite element program has been used to study: (i) unsymmetric boundary conditions and configurations such as oblique edge cracks in tension plates; (ii) general plane fracture problems with stress, displacement, or mixed conditions; (iii) curved cracks through the thickness of tension specimens; (iv) problems involving material inhomogeneity such as cracks intersecting bi-material interfaces in tension plates, wherein stress singularities of the general $r^{-\alpha}$ type may arise; (v) cracks meeting a free surface such as single-edge-notched tension plates with cracks of varying aspect ratio to study the effects of free surfaces on crack-tip stress intensity factors; (vi) cracks in orthotropic sheets, with the crack line located either symmetrically or unsymmetrically with respect to directions of principal

axes of orthotropy, and for various values of E_x/E_y to obtain stress intensity factors for laboratory test specimens. These results are documented in Refs. 5-8.

B. Study of Convergence of the Hybrid Element Procedure for Crack Problems.

A convergence study was conducted for the assumed displacement hybrid finite element method for solving stress intensity factors in linear fracture mechanics. Also methods to improve the satisfaction of stress-free conditions on a boundary segment of an element adjoining the crack surface were developed. It was found that a proper way to obtain convergent solutions for K-factors is to keep the size of the hybrid crack elements near the crack-tip at constant, pre-determined optimum size, while the size of the surrounding regular elements is progressively decreased. These results are documented in Refs. 1, 5, 6 and 7.

C. Three-Dimensional Hybrid Crack Element for Linear Elastic Fracture Analysis.

A finite element procedure was developed for the calculation of modes I, II and III stress intensity factors, which vary along an arbitrarily curved three-dimensional crack front in a structural component. The present finite element model is based on a modified variational principle of potential energy with relaxed continuity requirements for displacements at the inter-element boundary. The variational principle is a three-field principle, with the arbitrary interior displacements for the elements, interelement boundary displacements, and element boundary tractions as variables. The unknowns in the final algebraic system of equations, in the present model, are the nodal displacements and the three elastic stress intensity factors. Special elements, which contain proper square root and inverse square root variations in displacements and stresses, respectively, are used in a fixed

region near the crack front. Interelement displacement compatibility is satisfied, by assuming an independent interelement boundary displacement field, and using a Lagrangian multiplier technique to enforce such interelement compatibility. These Lagrangian multipliers, which are physically the boundary tractions, are assumed from an equilibrated stress field derived from three-dimensional Beltrami (or Maxwell-Morera) stress functions that are complete. However, considerable care had to be exercised in the use of these stress functions such that the stresses produced by any of these function components are not linearly dependent.

Since the method is based on a rigorous variational principle, which enforces at least on an average the conditions of interelement displacement continuity when \sqrt{r} type displacements are included in the near-tip region, the convergence of the finite element solution for nodal displacements as well as the stress intensity factors is established mathematically.

The geometry of the "basic element" used presently is a 20-node "isoparametric" brick element, with 60 degrees of freedom per element. Two options were developed: (a) a singular element for use near the crack front, wherein the stress intensity factors K_I , K_{II} and K_{III} are constant, and (b) a singular element where in the intensity factors K_I , K_{II} and K_{III} themselves vary quadratically. It is believed that this option will reduce the number of elements to be used near the crack front, in order to obtain an accurate variation of the intensity factors along the crack front.

The relevant matrices were evaluated numerically, using non-product type quadrature formulae, with proper mathematical transformations being used when singular type functions are encountered in stresses and strains in the near-tip region.

The mathematical development of the 3-D hybrid crack element and related computational details are documented in Refs. 2, 10 and 13.

D. Hybrid Finite Element Solutions of Fundamental Three-Dimensional Crack Problems.

The basic development for 3-D hybrid crack element discussed in (C) above has been exhaustively tested in analyzing various basic problems of 3-dimensional fracture that are of current interest. These include: (i) semi-elliptical surface flaws of various aspect ratios and various crack-depth ratios in plates under tension as well as bending; (ii) semi-elliptical surface flaws of various aspect ratios and various depth ratios on the inner and outer surface of pressurized thick shells; (iii) semi-elliptical surface and quarter-elliptical corner cracks near fastener holes in plates, typical of aircraft structural configurations. Moreover, the method was completely checked as to its accuracy and convergence in several other problems such as Sneddon's embedded penny shaped crack, buried elliptical cracks, and several other problems, for which analytical solutions exist. Also, the question of the optimum size of a singular element near the crack front has been studied thoroughly. All these results, along with the detailed mathematical derivations, have been presented in a recent Ph. D. thesis by Kathiresan [2], and are submitted to AFOSR for approval for publication as an AFOSR Technical Report [21]. The results for various problems have also been published in open literature [13, 17, 19, 24, 26, 28, 31, 32].

More recently, the 3-D hybrid crack element procedure has been applied to study the crack growth behaviour in adhesively bonded structure, wherein, each layer was treated in a three-dimensional fashion. Specifically results were obtained for a crack through the thickness of only the outer layer, near

a hole through the thickness of the entire stack of layers. These are believed to be the only fully three-dimensional analysis results for the problem under consideration; and are presented in Ref. [32] along with a discussion as to the validity of the obtained results.

E. Two-Dimensional Hybrid Element Procedure for Elastic-Plastic Analysis of Large Scale Yielding Fracture:

To study the problem of ductile fracture under large-scale yielding conditions, research was conducted in formulating and developing a second generation finite element procedure with the objectives: (i) developing circular-sector shaped embedded-singularity finite elements near the crack-tip. The correct r dependence of the dominant singular solution, corresponding to the nonlinear material model (Ramberg-Osgood law), was embedded in these near tip elements, where as the θ -dependence was approximated in each sector element and solved for in the sense of the finite element method; (ii) maintaining continuity of displacements and tractions, between near-tip elements with singular stress/strain assumptions and the far-field elements with regular stress/strain assumptions, through a hybrid displacement finite element model, (iii) using a J_2 -flow theory of plasticity and arbitrary kinematic hardening which will accurately model the Bauschinger effect under fully reversed and cyclic loading, (iv) using an incremental finite element solution procedure that will be suitable in the limiting case of elastic-perfect-plastic materials, as well as cyclic loading situations. The "initial-stress" iteration approach was used for this purpose, (v) developing a more accurate finite element method for incremental analysis of elastic-plastic problems. The more common approach in literature is to use "constant-stress" elements, and based on stress level in each element, the whole element either

yields or stays elastic. Thus in problems such as the present, where the yield zones near the crack-tip play a dominant role in the analysis and its interpretation, in order to obtain a reasonably accurate description of the yield zone, a very fine finite element mesh is needed. However, if higher-order elements are used, and if "plasticity-correction" iterations are performed at several points within the element, it then becomes possible to give a smoother definition of the yield zone. Thus, a portion of the element, in the present formulation, can yield while the rest of the element can remain elastic.

To start with, crack-tip blunting and other finite geometry changes near the crack-tip were ignored, and thus the analysis was restricted to a small-deformation assumption. This procedure was used to solve a hypothetical 3-point bend test specimen which was suggested, as a standard test for analytical solutions, by the ASTM Committee E-24 Round Robin Group on Elasto-Plastic Fracture Criteria. A detailed analysis of these results, with various fracture criteria in mind, was presented in (Atluri and Nakagaki [12], and Atluri and Nakagaki [14]). However, no other independent solution, from members of the Round Robin, with which to compare our results is available at the time of this writing.

F. Study of the J-integral as a Fracture Initiation Criterion Under Large-Scale Yielding.

The above small-deformation analysis procedure was used to analyze a compact tension cracked specimen, of A533B steel, for which experimental fracture data was reported by the Westinghouse Research Laboratories. The thickness of the experimental specimen was roughly $1/2$ - $1/3$ of the characteristic inplane dimensions. Thus it was not a priori certain whether the conditions

near the crack-tip can be characterized mathematically as those of plane stress or of plane strain. Thus the problem was analyzed using two types of two-dimensional approximations. Also, the experimental uniaxial stress/strain curve supplied to us by the Westinghouse Laboratories was seen to possess a yield point instability, which the numerical procedure could not account for. Thus, two different, smooth, Ramberg-Osgood type curves were used to fit the actual experimental stress-strain curve. From these analyses, it was found that: (i) the mathematical characterization of the material-property data had no significant effect on the results, (ii) the empirical formula of Rice et al. for estimating J was in significant error in the case of the compact tension specimen, and (iii) the computed plane-stress values for J were much closer to the experimental data, as compared to the plane-strain values which differed by a factor of about 3. Thus it was concluded that the plane-stress conditions more accurately characterize the plastic flow near the crack-tip in "small" specimens. These results are documented in detail in (Atluri and Nakagaki [15]).

However, even the computed plane-stress values for J , based on the present analysis with a "non-growing stationary crack" assumption, differed by about 15-20% from the cited experimental results for J at fracture. From subsequent private communications with Begley and Landes, who conducted the experiments, it was learned that, in fact, there may have been stable crack growth, which was not monitored, in the experiments.

Taking into account the effect of this stable crack growth, and the results of a refined and consistent finite deformation analysis of the problem, as discussed in (G) below, it was concluded that J is a computationally advantageous criterion for the onset of crack growth in ductile materials.

G. Studies of Finite Deformation Effects near the Crack-tip.

To delineate the effects of crack-tip blunting, and the attendant finite geometry changes near the crack-tip, under large scale plastic yielding conditions, a finite deformation, embedded singularity, elastic plastic incremental finite element procedure was developed [16]. The test problem was that of a 3-point bend fracture test specimen, the experimental data for which was reported and made available to us by Westinghouse. A detailed comparison of the results, for stresses, strains, displacements, and J, from this finite deformation analysis, and those obtained using the above small-deformation analysis was made. Significant conclusions of the above comparison were: (i) not with-standing the crack-tip blunting, J still remains as a valid parameter to characterize the severity of the near-tip condition, (ii) since the well-known Rice's definition of J is invalid for finite-deformation, a modified definition must be derived. This new definition involves the strain energy density function that is dependent on Piola-Lagrange (unsymmetric) stress tensor, and the traction term appearing in the J-integral should be interpreted as Piola-Lagrange tractions in the deformed geometry as referred to the undeformed configuration. When this modified definition for J is employed, a more accurate path-dependence (within $\pm 1.5\%$) was noted for J, as compared to the small-deformation case wherein the J value on paths closest to the crack-tip was about 15% lower than that on paths in the far field. A detailed discussion of these results was presented in (Atluri, Nakagaki, and Chen [16]).

One interesting observation was that at all load-point displacement levels, including that at fracture, there was an excellent correlation (within $\pm 4\%$) between the presently computed results for J and those in the Westinghouse experiments. This suggested that no appreciable stable crack growth may

have been present in the experiment. Subsequent private communications with Dr. Landes at Westinghouse appeared to confirm this. Thus because of the particular specimen geometry and loading conditions, 3-point bend specimen may offer the advantage of precisely measuring J_{IC} for crack growth initiation for a given ductile material.

Further refinements, such as the incorporation of a "knee-correction", to achieve better convergence of plastically adjusted stresses, and a more accurate incremental elastic-plastic law, were made in the above finite deformation analysis. The details of the nature and form of singularity of actually computed strains and stresses, and their variation in the angular coordinate near the crack tip were critically reviewed and compared with the small-deformation analysis, using J_2 deformation plasticity theory, of Hutchinson, Rice and Rosengren. These results are discussed in detail in (Atluri and Nakagaki [20]). Because of the finite root radius of the blunted notch, there can only be a strain concentration, however large, at the tip of the notch. The effect of this strain concentration, as opposed to the strain-singularity at the blunted crack tip, was properly accounted for in the analysis of [20].

The refined finite-deformation analysis procedure was also used to analyze three fracture test specimens, of 3-point bend, center-cracked, and compact tension types, of configurations identical to those in the experiments of Westinghouse Laboratories. The obtained numerical data was analyzed with both the J and COD criteria in mind, and the following conclusions were drawn: (i) the computed J correlated well with the cited experimental results for all the specimens; (ii) J was found to be directly correlated with COD for these test specimens. The above data indicated that, for these specimens, $J = 1.44\sigma_y$ (COD) for moderately hardening materials, where

σ_y is the yield stress. Thus it can be seen that COD may also be used as a fracture criterion; (iii) however, it was found that the computation of COD, as essentially a near-tip geometric quantity, for arbitrary plane problems of ductile fracture, is not well defined. Attempts to correlate COD with such near-tip geometrical quantities such as the crack-opening at points where the elastic-plastic boundary intersects the crack-profile were found to be discouraging; (iv) the empirical formula of Rice et al. for J was once again found to be inaccurate for the compact tension specimens. These results are elaborated upon in (Atluri, Nakagaki, and Chen [18]).

Once again, the computed J for the compact tension specimen, based on the finite element modeling of a non-growing crack, were found to be about 10-15% less than in the experiments. Again this suggested that there may, in fact, have been a subcritical crack growth prior to fracturing in the experiments, thus providing motivation for our study (H) on stable crack growth, described below.

H. Analysis of Stable Crack-Growth and Study of Criteria Governing Loss of Stability.

A finite element methodology was developed to study the phenomena of stable crack growth in two-dimensional problems involving ductile materials. Crack growth is simulated by (i) the translation in steps, of a core of sector elements, with embedded singularities of Hutchinson-Rice-Rosengran type by an arbitrary amount, Δa in each step, in the desired direction; (ii) reinterpolation of the requisite data in the new finite element mesh; and (iii) incremental relaxation of tractions in order to create a new crack face of length Δa . Steps (i) and (ii) were followed by corrective equilibrium-check iterations. A finite deformation analysis, based on the incremental

updated Lagrangean formulation of the hybrid-displacement finite element method, is used.

The present procedure is used to simulate available experimental data on stable crack growth, and thus study the variation, during crack growth, of certain physical parameters that may govern the stability of such growth and the subsequent onset of rapid fracture. Attention is focussed in this study on the parameters: $G^{*\Delta}$ the energy release to the crack-tip per unit crack growth, for growth in finite steps Δa , calculated from global energy balance considerations; G_{pz}^{Δ} the energy release to a finite "process zone" near the crack-tip per unit crack growth, for growth in finite steps Δa , calculated again from global energy balance considerations; and the crack opening angles.

However, only the first phase of our study, viz., the simulation of available experimental data, has been completed.

It is recognized that formulating any criterion or criteria, governing the loss of stability of crack-growth, based on numerical simulation of a few experimental data is, at best, a risky proposition. Thus, we defer any conclusions regarding such criteria until the completion of second phase of our research, to be conducted. However, the results obtained so far, can lead to the following conclusions that may be germane to the problem of stable crack growth in ductile materials: 1) A direct numerical proof is provided for the original hypothesis of Rice that $G^{*\Delta} \rightarrow 0$ as $\Delta a \rightarrow 0$ for those materials for which the flow stress saturates at a finite value of large strain. Thus, for any meaningful numerical study of stable crack growth, a finite growth step must be postulated. Results obtained during this research and reported in [27,30] may be useful in providing guidelines for choosing Δa such that the numerically computed $G^{*\Delta}$ is not sensitive to the errors inherent in numerical processes such as in the finite element method. 2) Since the

magnitudes of $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ are clearly shown [27, 30] to depend on the postulated magnitudes of "finite" growth steps Δa , in the finite element modeling, it is clear that any criteria governing the loss of stability of crack growth cannot be based on the absolute magnitudes of these quantities. Any such criteria can only be based on the relative qualitative behaviour of $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ for the postulated growth step Δa . 3) In view of the previous observation and the fact $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ vary substantially during the crack extension process, it is clear that the generalizations of Griffith's approach, in the sense that a ductile material has some characteristic work of separation, per unit new crack area, and that this is to be equated at the critical condition to the rate of surplus of work done on the material, cannot be made in situations of stable growth under large-scale yield conditions. 4) The only discernible trends near the points of fracture as observed in experiment, for both the cases studied, is the marked change in the behaviour of $G^{*\Delta}$, CTOA, and to an extent in $G_{\Gamma}^{*\Delta}$, near these points, when these quantities reverse their monotonically increasing trend during the prior extension process. While a theoretical argument explaining this is lacking, it yet remains to be seen whether these observations can be used to numerically predict loss of stability of growth in different specimens of the same material. This is the object of our work yet to be conducted.

A detailed discussion of the present analysis of stable crack growth and results leading to the above conclusions, have been presented in (Atluri and Nakagaki [27], and Nakagaki, Chen, and Atluri [30]).

I. Analysis of Effects of Crack-Closure on Fatigue Crack Growth Rates.

The crack growth simulation procedure described in (G) above was also used to study the crack-closure and-opening stresses under Mode I type

constant amplitude fatigue loading. The values of closure and opening stresses for a center cracked Aluminum panel subject to constant amplitude cyclic loading, which causes a "small-scale" yielding near the crack-tip, were found to correlate well with existing results in literature for identical problems. However, the number of finite element degrees of freedom used in the present study was found to be an order of magnitude lower than in current literature for comparable accuracy. These results are yet to be published [30].

Considerations of general spectrum loading to understand the crack growth acceleration and retardation effects still remains and is the object of our research yet to be conducted.

4. CONCLUDING REMARKS AND ACKNOWLEDGEMENTS

The writer wishes to take this opportunity to express his deep sense of appreciation to his associates in this research; Drs. Mich Nakagaki, Krish Kathiresan, and Wen-Hwa Chen, for their significant contributions to the progress of the above reported research.

He also sincerely thanks the AFOSR Program Manager, Dr. William J. Walker, who provided timely encouragement and also insights into the real world problems of fracture mechanics relating to aerospace structures.

5. PUBLICATIONS

5.1 GRADUATE THESES

1. M. Nakagaki, "Displacement Hybrid Finite Element Method for 2-D Crack Problems" Ph. D. Thesis, University of Washington, Seattle, 1975.
2. K. Kathiresan, "Three-Dimensional Linear Elastic Finite Element Model", Ph. D. Thesis, Georgia Institute of Technology, 1976.
3. W. H. Chen, "Finite Deformation, Elastic-Plastic Finite Element Analysis of Planar Cracks", Ph. D. Thesis, Georgia Institute of Technology, 1977.

5.2 PAPERS IN JOURNALS AND PROCEEDINGS

4. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "Application of an Assumed Displacement Hybrid Finite Element Procedure to Two Dimensional Problems of Fracture Mechanics", AIAA paper No. 74-390 in Proceedings of AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974.
5. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "An Assumed Displacement Hybrid Finite Element Model for Linear Fracture Mechanics", International Journal of Fracture, Vol. 11, No. 2, pp. 257-271, April 1975; Also a summary appears in Proceedings of 7th U. S. National Congress of Applied Mechanics, ASME, March 1975, p. 104.
6. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "Fracture Mechanics Application of an Assumed Displacement Hybrid Finite Element Procedure", AIAA Journal, Vol. 13, No. 6, pp. 734-740, June 1975.
7. S. N. Atluri, A. S. Kobayashi, and M. Nakagaki, "A Finite Element Program for Fracture Mechanics Analysis of Composite Materials", Fracture of Composites, ASTM Special Technical Publication, 593, American Society for Testing and Materials, pp. 86-98, November 1975.
8. S. N. Atluri, "On Hybrid Finite Element Models in Solid Mechanics", Advances in Computer Methods for Partial Differential Equations, (R. Vishnevetsky, Editor) Rutgers University Press, pp. 351-362, June 1975.
9. S. N. Atluri, "Finite Element Perturbation Analysis of Nonlinear Dynamic Response of Elastic Continua", Finite Element Methods in Engineering, Proceedings of 1974 International Conference on Finite Element Methods in Engineering (Pulmano and Kabaila, Editors), Unisearch, Ltd. Sydney, Australia, pp. 521-538, September 1974.
10. S. N. Atluri, K. Kathiresan, and A. S. Kobayashi, "Three-Dimensional Linear Fracture Mechanics Analysis by a Displacement Hybrid Finite Element Procedure", paper No. L-7/3, Transactions of the Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975.

11. S. N. Atluri, A. S. Kobayashi, and J. S. Chang, "Brain-Tissue Fragility: A Finite Strain Analysis by a Hybrid Finite Element Method", Journal of Applied Mechanics, Trans. ASME, Series E, Vol. 42, No. 2, pp. 269-274, June 1975.
12. S. N. Atluri, and M. Nakagaki, "Analysis of Two-Dimensional Fracture Problems Involving Large-Scale Yielding: A Displacement Hybrid Finite Element Method", Proceedings of 12th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, pp. 77-87, October 1975.
13. S. N. Atluri, and K. Kathiresan, "An Assumed Displacement Hybrid Finite Element Model for Three-Dimensional Linear Fracture Mechanics Analysis", Proceedings of 12th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, pp. 66-76, October 1975.
14. S. N. Atluri, and M. Nakagaki, "Post-Yield Analysis of a Three-point Bend Fracture Test Specimen: An Embedded Singularity Finite Element Incremental Method", Developments in Theoretical and Applied Mechanics, Vol. 8, pp. 265-283, VPI and SU Press, April 1976.
15. S. N. Atluri, and M. Nakagaki, "J-Integral Estimates for Strain-Hardening Materials in Ductile Fracture Problems", Proceedings of AIAA/ASME/SAE 17th Structures, Structural Dynamics and Materials Conference, Valley Forge, Pennsylvania, May 1976, pp. 140-152.
16. S. N. Atluri, M. Nakagaki, and W. H. Chen, "Fracture Analysis Under Large-Scale Yielding: A Finite-Deformation, Embedded Singularity, Elasto-Plastic Incremental Finite Element Solution", in Flaw Growth and Fracture; ASTM STP 631, American Society for Testing and Materials, August 1977, pp. 43-61.
17. S. N. Atluri, K. Kathiresan, and M. Nakagaki, "Inner Surface Cracks in an Internally Pressurized Cylinder Analyzed by a Three-Dimensional Displacement Hybrid Finite Element Method", In Pressure Vessel Technology, (Part II, Materials and Fabrication), ASME, New York, NY, April 1977, pp. 527-535.
18. S. N. Atluri, M. Nakagaki, and W. H. Chen, "Fracture-Initiation in Plane Ductile Fracture Problems: An Elastic-Plastic Finite Element Incremental Analysis," Pressure Vessel Technology, (Part II, Materials and Fabrication), ASME, April 1977, pp. 579-593.
19. S. N. Atluri, and K. Kathiresan, "On A 3-D Singularity Element for Computation of Mixed-Mode Stress Intensities", in NASA CP-2001, pp. 267-275 (November 1976), Proceedings of 13th Annual Meeting of Society of Engineering Science and also in abstract form in "Three-Dimensional Fracture Analysis", (Ed. by L. E. Hilbert, Proceedings of a Workshop held at Battelle Columbus Labs, April 1976), pp. 11-14.
20. S. N. Atluri, and M. Nakagaki, "Stress Analysis of Cracks in Elasto-plastic Range," Fracture 1977, Vol. 3, (D. M. R. Taplin, Editor), University of Waterloo Press, June 1977, pp. 457-463.

21. S. N. Atluri, and K. Kathiresan, "Hybrid Crack Element For 3-D Fracture Analysis" submitted to AFOSR for approval as AFOSR-TR, 1977.
22. S. N. Atluri, and M. Nakagaki, "J-Integral Estimates for Strain Hardening Materials," AIAA Journal, Vol. 15, No. 7, July 1977, pp. 842-851.
23. J. S. Cheng, S. N. Atluri, and A. S. Kobayashi, "Hybrid Displacement Finite Element Method For Nonlinear Elastic and Elastic-Plastic Analyses", Proceedings of 18th AIAA/ASME/SAE Structures, Structural Dynamics and Materials Meeting, San Diego, California, March 1977, pp. 279-290.
24. S. N. Atluri, M. Nakagaki, K. Kathiresan, and H. C. Rhee, "Fracture Analysis of Structures Under Combined Mode Loadings", in Advances in Civil Engineering through Engineering Mechanics (Proceedings of 2nd ASCE EMD Specialty Conference, Rayleigh, NC, May 1977) ASCE, New York, NY, May 1977, pp. 89-93.
25. S. N. Atluri, "Hybrid Crack Elements," Proceedings of Joint AFOSR/AFRPL Conference on Propellants, Edwards Air Force Base, California, March 1974 (Extended abstract only).
26. S. N. Atluri, and K. Kathiresan, "Outer and Inner Surface Flaws in Thick-Walled Pressure Vessels," Paper G 5/4, Transactions of 4th International Conference on Structural Mechanics in Reactor Technology, August 1977, 13 pages.
27. S. N. Atluri and M. Nakagaki, "An Analysis of Stable Crack Growth in Ductile Materials", in Materials and Processes - In Service Performance, National SAMPE Technical Conference Series, Vol. 9, October 1977, pp. 48-57.
28. K. Kathiresan, and S. N. Atluri, "Surface Flaws in Thick Plates," in Proceedings of 14th Annual Society of Engineering Science Meeting, Lehigh University, Bethlehem, PA, November 1977, pp. 351-358.
29. S. N. Atluri, M. Nakagaki, K. Kathiresan, H. C. Rhee, and W. H. Chen, "On Hybrid Finite Element Models for Linear and Nonlinear Fracture Analyses", in Numerical Methods in Fracture Mechanics (Editors: Luxmore and Owen) Proceedings of International Conference on Numerical Methods in Fracture Mechanics, Swansea, UK, January 9-13, 1978) pp. 52-67, 1978.
30. M. Nakagaki, W. H. Chen, and S. N. Atluri, "A Finite Element Analysis of Stable Crack Growth: Formulation of Criteria", to appear in an ASTM STP, Proceedings of ASTM National Symposium on Elastic-Plastic Fracture, Atlanta, GA, November 1977.
31. S. N. Atluri and K. Kathiresan, "Surface Flaws in Pressure Vessels", Invited paper, Nuclear Engineering and Design, 1978. (to appear).
32. S. N. Atluri and K. Kathiresan, "Stress Analysis of Typical Flaws in Aerospace Structural Components Using a Three-Dimensional Hybrid Element Procedure", in Proceedings of 19th AIAA/ASME/SAE Structure, Structural Dynamics and Materials Conference, Bethesda, MD, April 3-8, 1978.

33. S. N. Atluri, "A Review of Numerical Methods for Engineering Fracture Analysis", to be presented and published at 2nd International Conference on Fracture Mechanics, Bangalore, India, March 1979.

5.3 PAPERS PRESENTED AT PROFESSIONAL MEETINGS

34. AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Nevada, April 1974, paper title same as cited in Ref. 1.

35. 7th U.S. National Congress of Applied Mechanics, Boulder, Colorado, June 1974, paper title same as cited in Ref. 2.

36. 1974 International Conference on Finite Element Methods in Engineering, University of New South Wales, Sydney, Australia, August 29-31, 1974, paper title same as cited in Ref. 6.

37. NATO Advanced Study Institute on Continuum Mechanics Aspects of Rock Fracture and Geodynamics, Reykjavik, Iceland, August 1974, presented an invited paper, "Finite Element Analysis of Cracks Between Dissimilar Media".

38. ASTM Symposium on Fracture of High Modulus Fibers and Their Composites, National Bureau of Standards, Gaithersburg, MD, paper title same as cited in Ref. 4.

39. Conference on Fundamental Aspects of Deformation and Fracture of Composite Materials, Battelle Seattle Research Center, Seattle, Washington, February 1975, presented an invited paper, "Stress Intensity Factors of Cracked Orthotropic Plates", (Presented by A. S. Kobayashi).

40. ASME Joint Western and National Applied Mechanics Conference, University of Hawaii, Honolulu, March 1975, paper title same as cited in Ref. 8. (Presented by A. S. Kobayashi).

41. AICA International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, PA, June 1975, paper the same title as cited in Ref. 5.

42. AFRPL/Edwards AFB Contractors Meeting, California Institute of Technology, Pasadena, California, May 1975, invited presentation on "3-D Cracked Elements".

43. ASTM Committee E-24 Meeting, 9th U.S. National Symposium on Fracture Mechanics, Pittsburgh, PA, August 1975, invited presentation, "Analysis of Large-Scale Yielding Fracture Problems".

44. Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975, paper title same as cited in Ref. 7.

45.-46. 10th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, October 1975, paper titles same as cited in Ref. 9 and 10.

47. "Workshop on 3-D Fracture Analysis", Battelle Columbus Lab., Columbus, Ohio, April 1976, invited presentation on "3-D Hybrid Cracked Elements".

48. 8th South Eastern Conference on Theoretical and Applied Mechanics, VPI and SU, Blacksburg, VA, April 1976, paper title same as cited in Ref. 11.

49. 17th AIAA/ASME SDM Conference, Valley Forge, PA, May 1976, paper title same as cited in Ref. 12.

50. 10th U.S. National Symposium on Fracture, Philadelphia, August 1976, title same as cited in Ref. 13.

51. 13th Annual Meeting of Society of Engineering Science, NASA-Langley, Hampton, VA, November 1976. (Ref. 16).

52. 18th AIAA/ASME/SAE SDM Conference, San Diego, California, March 1977, title same as cited in Ref. 22, (presented by J. S. Cheng).

53. "Crack Elements for Bi-Material Fracture," JANNAP Propulsion Conference, Edwards Air Force Base, CA, March 1977.

54. "Inner Surface Flaws in a Thick Walled Pressure Vessel," 3rd International Conference on Pressure Vessel Technology, ASME/JSME, Tokyo, Japan, April 1977.

55. "Fracture Initiation in Plane Ductile Fracture Problems", 3rd International Conference on Pressure Vessel Technology, ASME/JSME, Tokyo, Japan, April 1977.

56. "Fracture Analysis Under Mixed Mode Conditions", 2nd ASCE Engineering Mechanics Specialty Conference, Raleigh, NC, May 1977.

57. "Stress Analysis of Cracks in Elasto-Plastic Range", 4th International Conference on Fracture, University of Waterloo, Canada, June 1977.

58. "Outer Surface Flaws in Pressure Vessels," 4th International Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, August 1977.

59. "Analysis of Stable Crack Growth in Ductile Materials", 9th SAMPE National Conference, Atlanta, GA October 1977.

60. "Criteria for Stable Crack Growth", ASTM National Symposium on Elastic-Plastic Fracture, Atlanta, GA November 1977.

61. "Surface Flaws in Plates", 14th SES Conference, Lehigh University, Bethlehem, PA November 1977.

62. "Stress Analysis of Typical Flaws in Aerospace Structural Components", 19th AIAA/ASME, Structures, Structural Dynamics Conference, Bethesda, MD April 1978.

63. "Finite Element Analysis of Elastic-Plastic 3-D Cracks", Joint AFOSR/AFRPL Rocket Propulsion Research Meeting, Edwards Air Force Base, April 1978.

5.4 SEMINARS AT UNIVERSITIES

1. "On Elastic-Plastic Fracture Mechanics", at Department of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, August 21, 1974.

2. "Computational Methods in Fracture Mechanics", National Aeronautical Lab, Bangalore, India, August 22, 1974.

3. "Computational Fracture Mechanics", College of Engineering, Boston University, Boston, Mass., January 1976.

4. "Hybrid Finite Elements for Fracture Analysis", four lectures at Short Course on Advances in Finite Element Methods, University of Tennessee Space Institute, Tullahoma, TN March 1978.

5. "3-D Crack Analysis by Finite Elements", University College, Cork, Ireland, January 14, 1978.

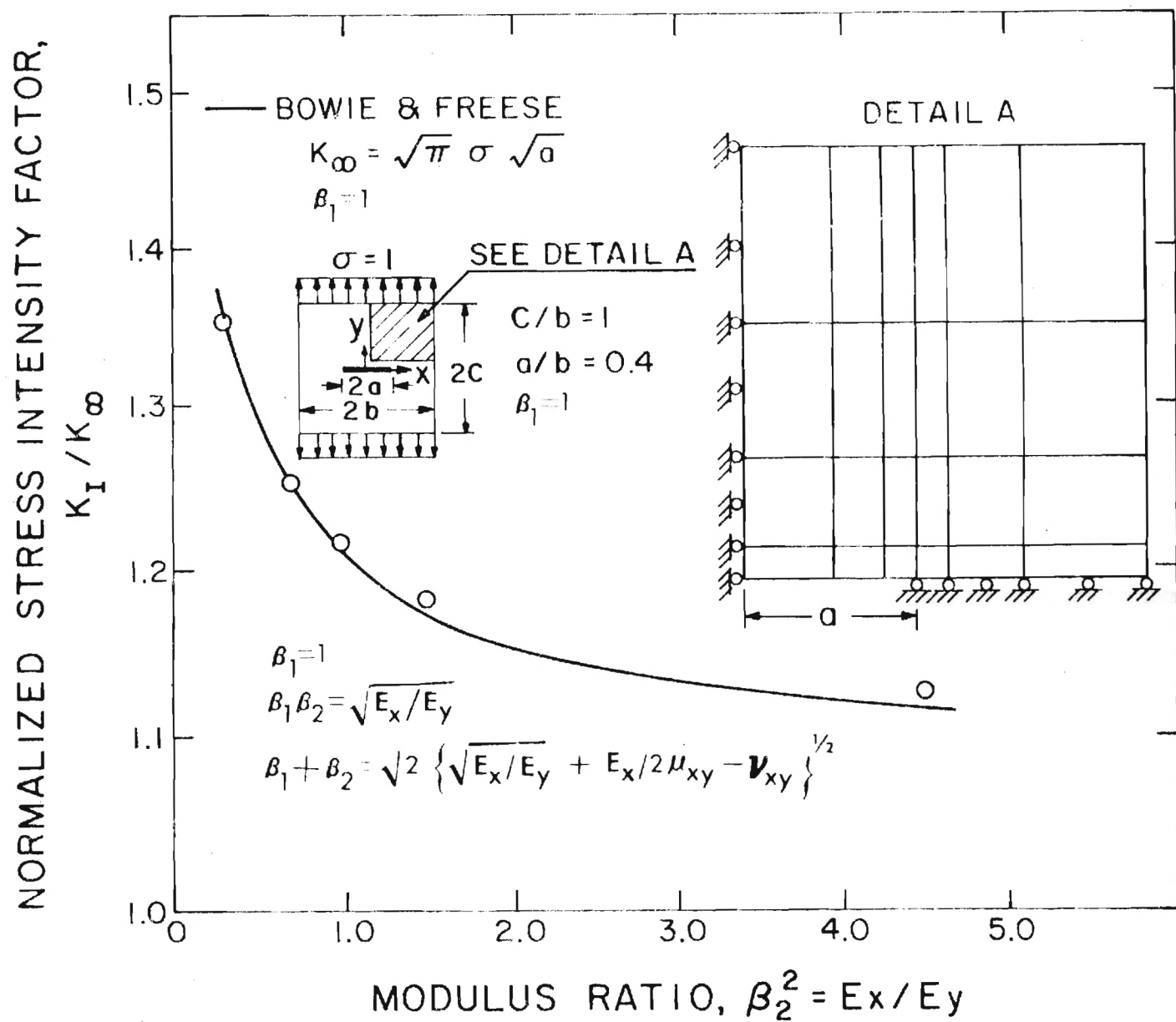


FIG. 1 (Taken From Ref. 7): Typical Result for a center cracked orthotropic sheet; crack symmetric w.r.t axes of orthotropy.

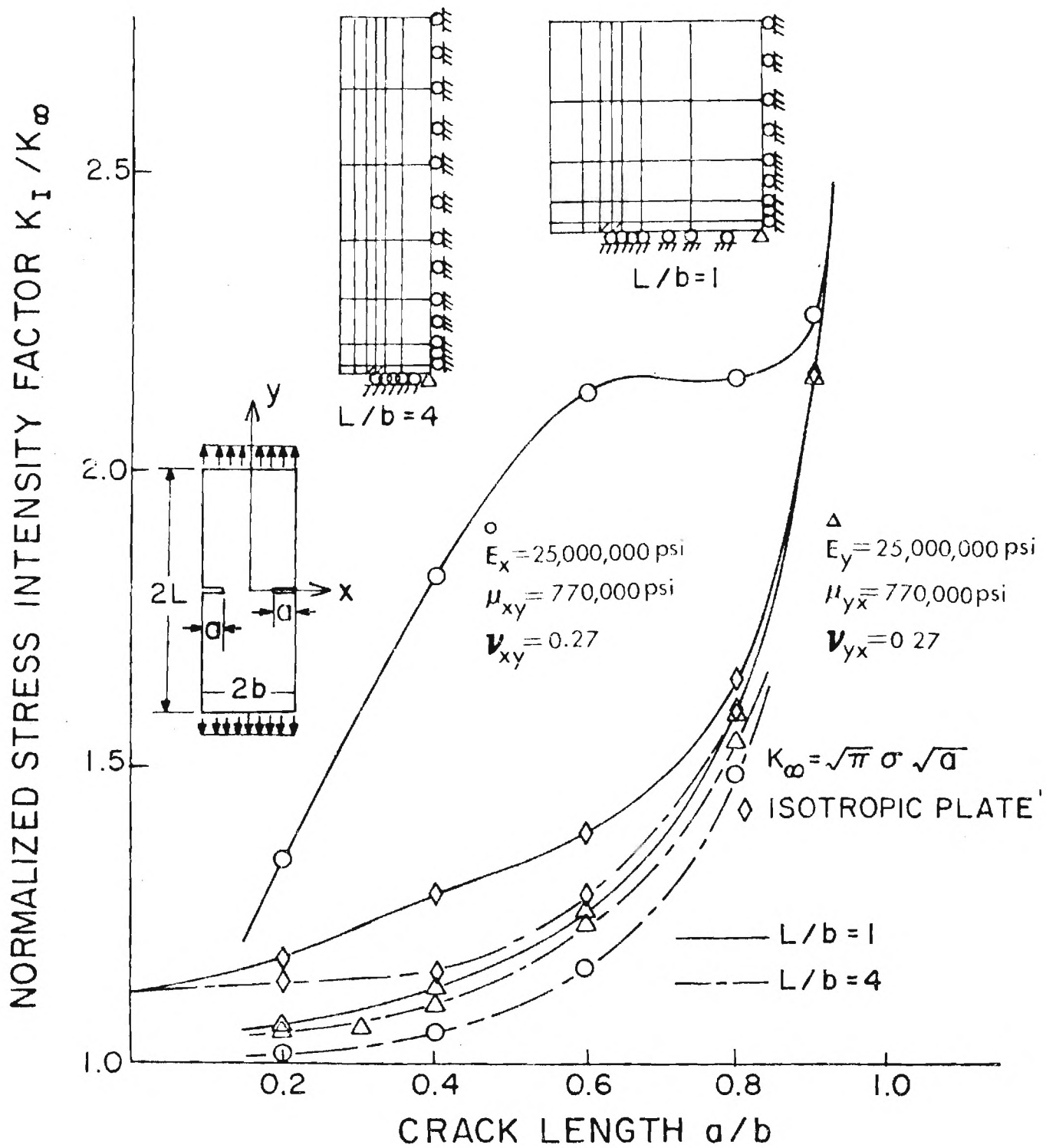


FIG. 2 (Taken From Ref. 7): Typical Result for an edge-cracked orthotropic sheet; crack symmetric w.r.t axes of orthotropy; Results for various Laboratory specimens.

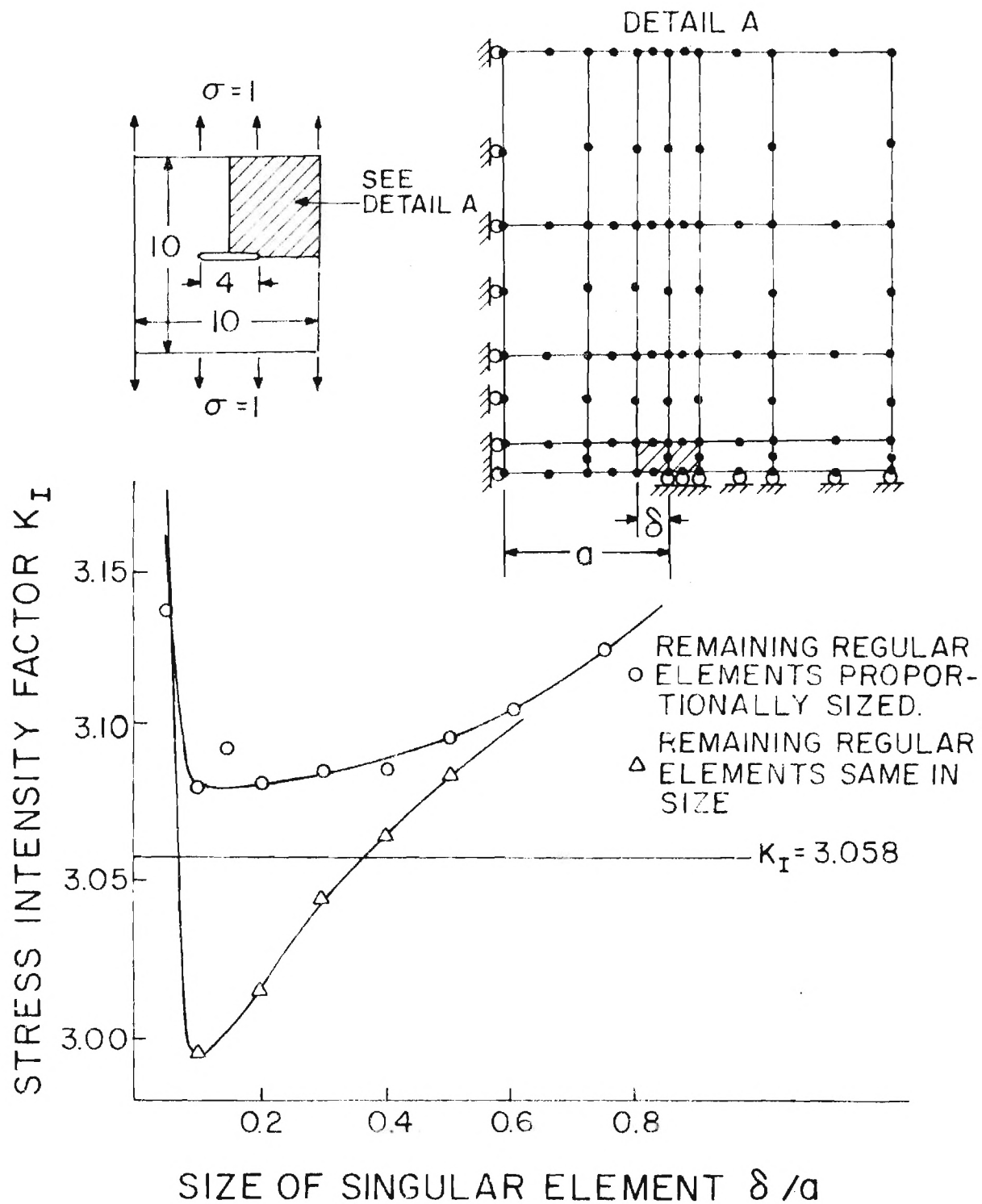


FIG. 3 (From Ref. 6): A Typical convergence study.

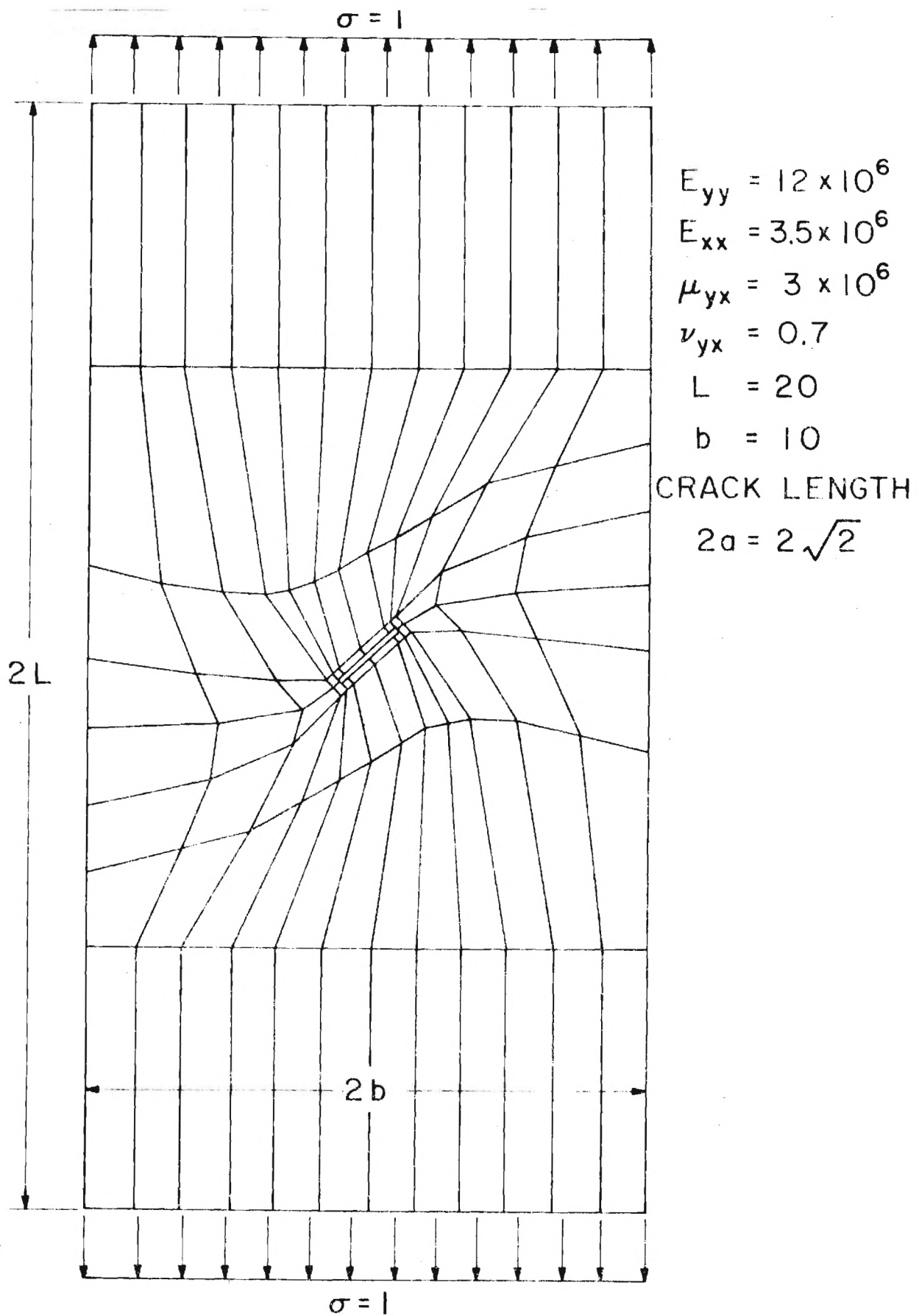


FIG. 4 (From Ref. 6): A Typical Mesh for analysing the problem of a crack unsymmetrically located w.r.t axes of orthotropy.

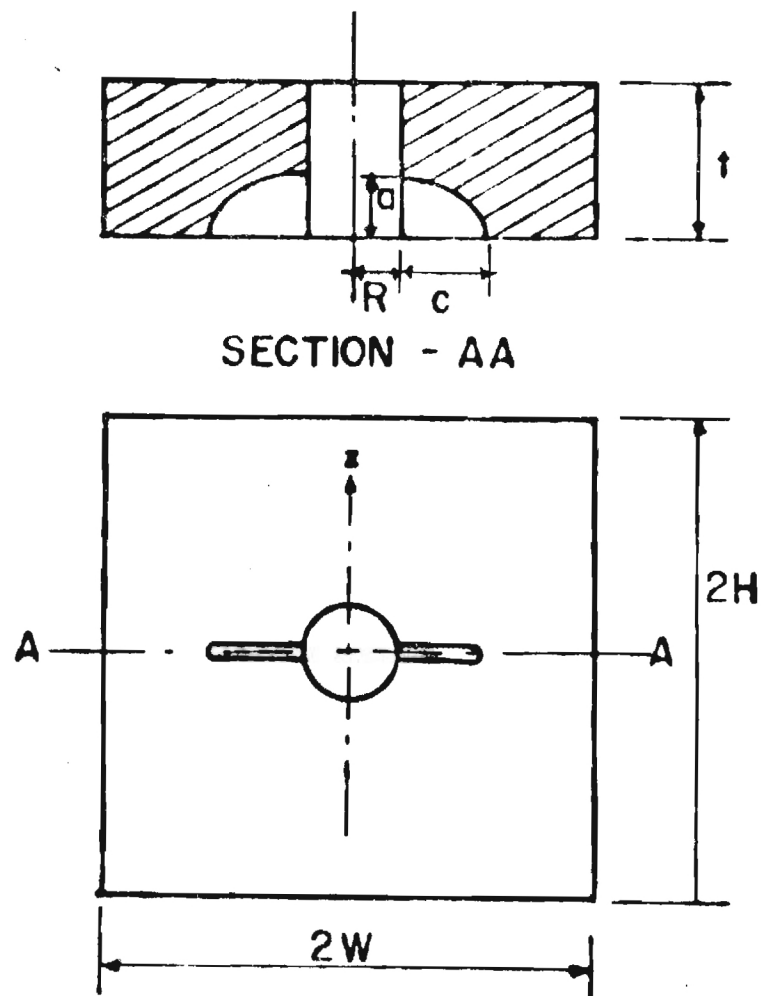
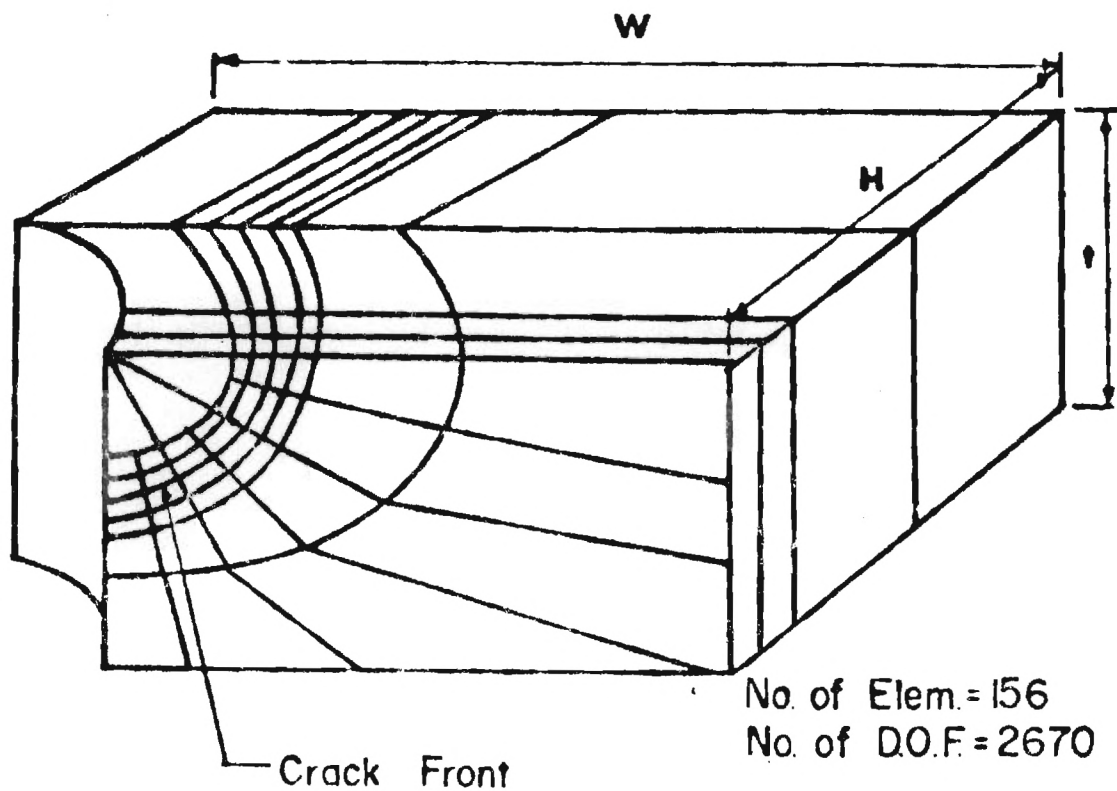


FIG. 45: Corner Cracks Emanating from Holes in Plates

FIG. 5 (From Refs. 2 and 21): A Typical 3-D Crack Problem.



**FIG. 46: Finite Element Breakdown of
Quarter of the Corner Cracked
Hole Problem**

FIG. 6 (From Ref. 21 and 32): Typical Finite Element Mesh for Problem in Fig. 5.

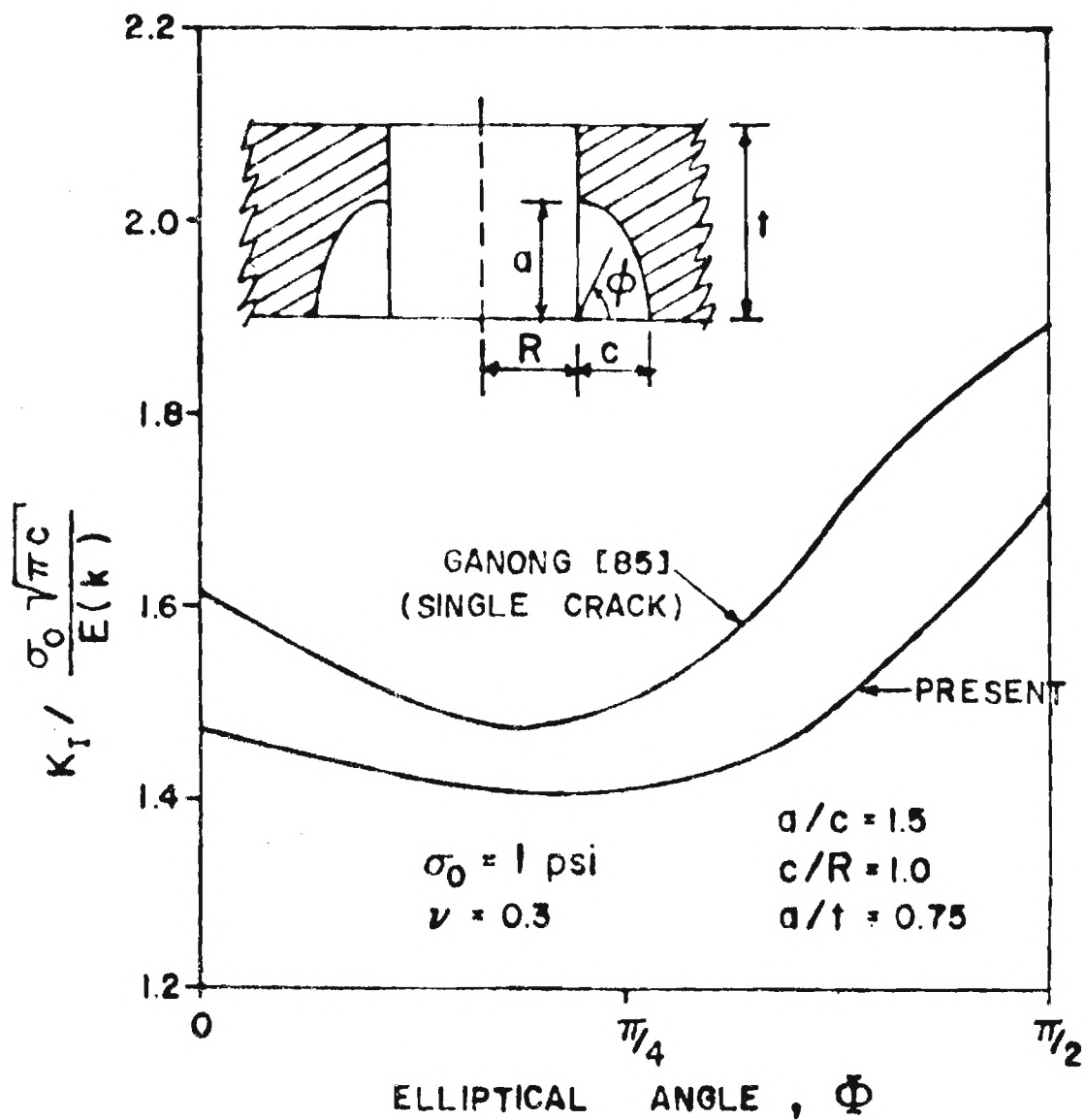


FIG. 5l: Variation of Stress Intensity Factor
 for Corner Cracks Emanating from
 Hole
 ($a/c = 1.5$; $c/R = 1.0$; $a/t = 0.75$)

FIG. 7 (Taken From Ref. 32): Typical Result for a corner crack near a
 Fastener Hole.

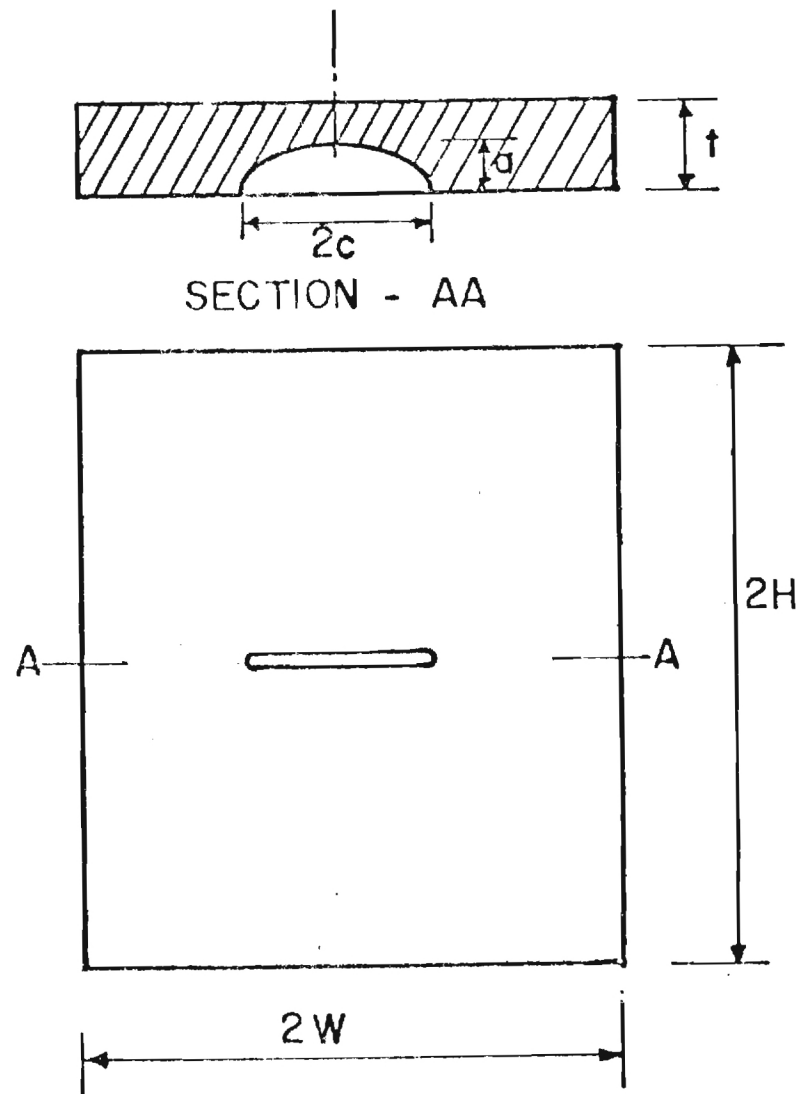
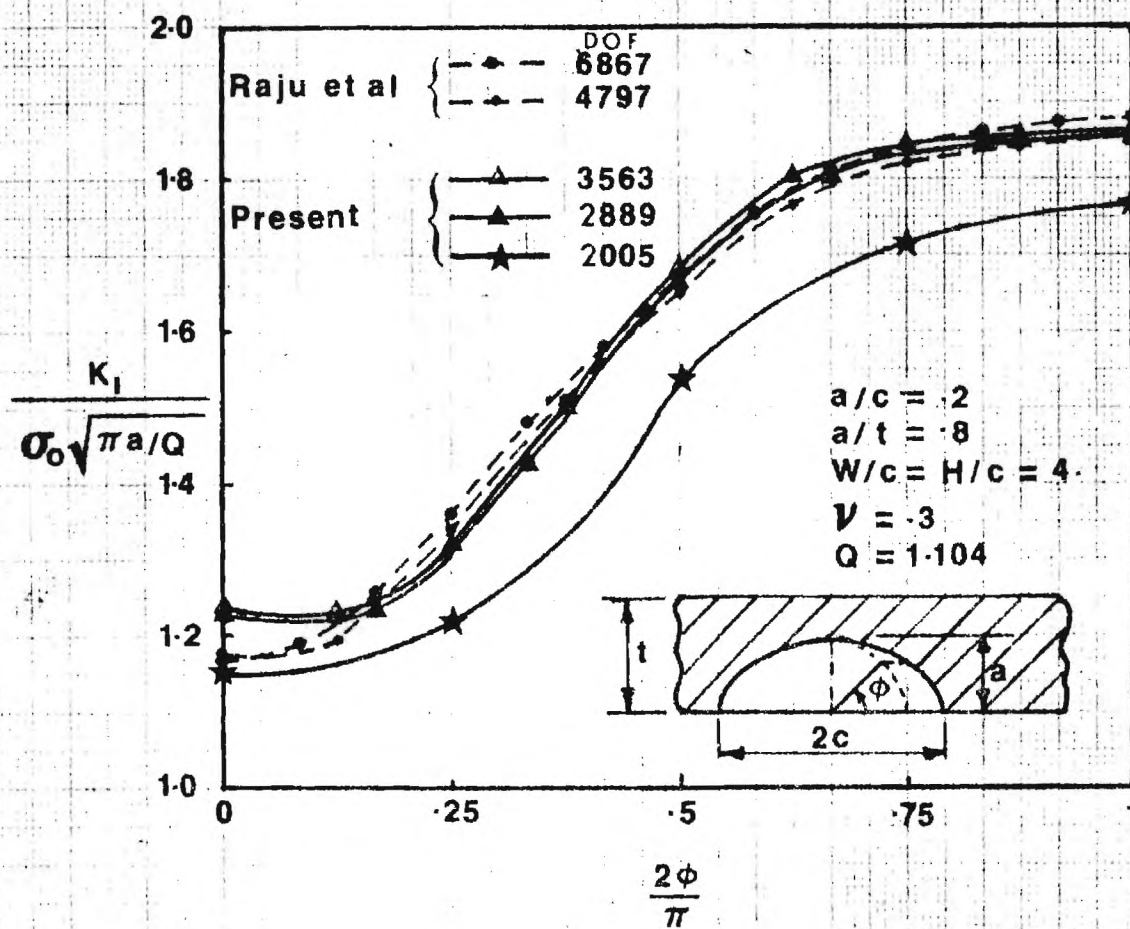


FIG. 2. Semi-Elliptical Surface Flaws in Plates in Tension and Bending

FIG. 8 (Taken From Ref. 28): A Typical 3 crack Problem of a surface flawed plate.



Variation of Stress Intensity Factors for a Semi-Elliptical Surface Crack in a Thin Plate ($a/t = .8$)

FIG. 9 (Taken from Ref. 33): A Typical 3-D Results for surface-flawed Tension Plate: Comparison of Hybrid Element Results with an alternate Finite Element Result.

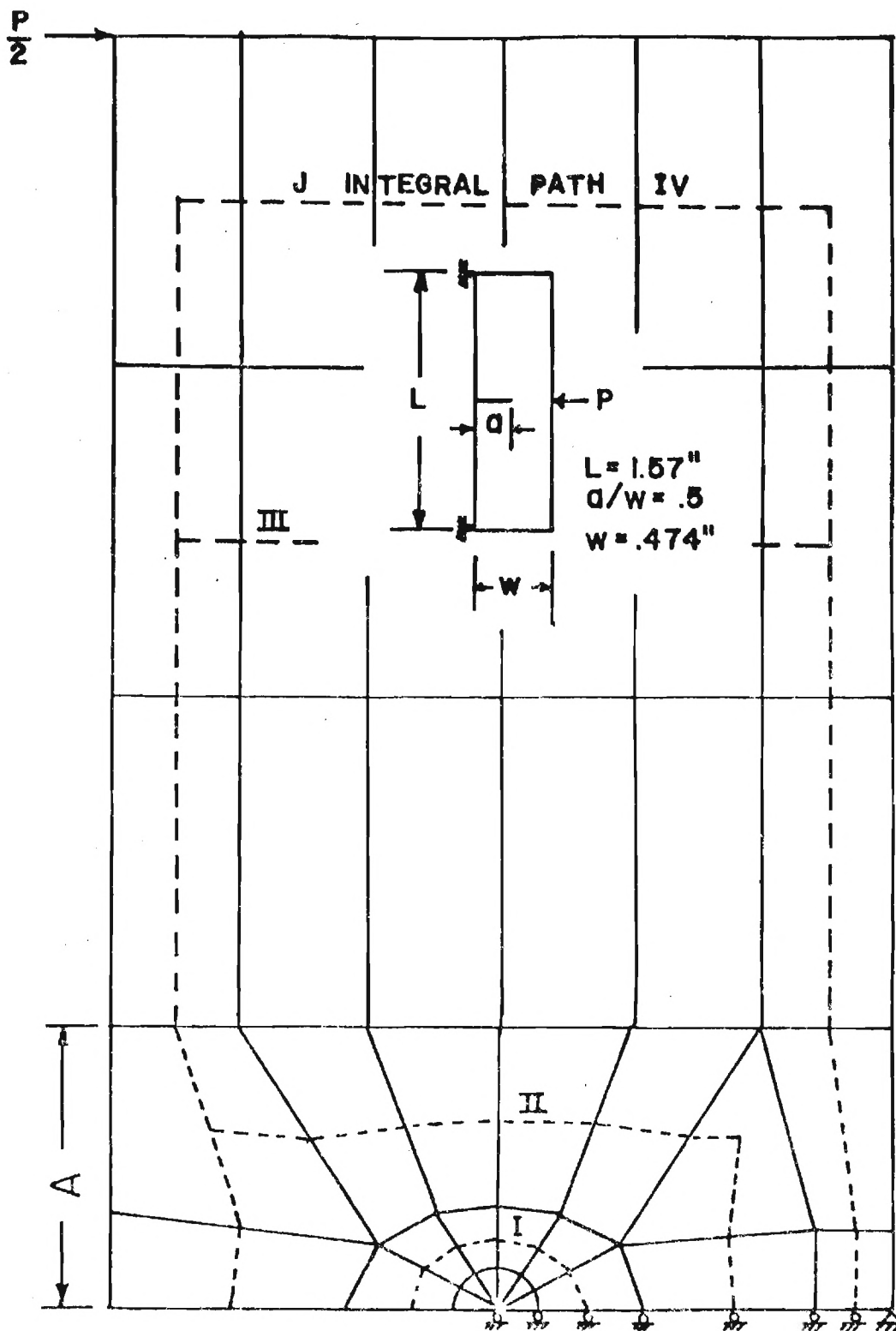


FIG. 2: Finite-Element Model of 3-Point Bend Bar (shown in insert) and J-Integral Paths

FIG. 10 (Taken From Ref. 16): A Typical Specimen: Analysis of Large-Scale Yielding Fracture.

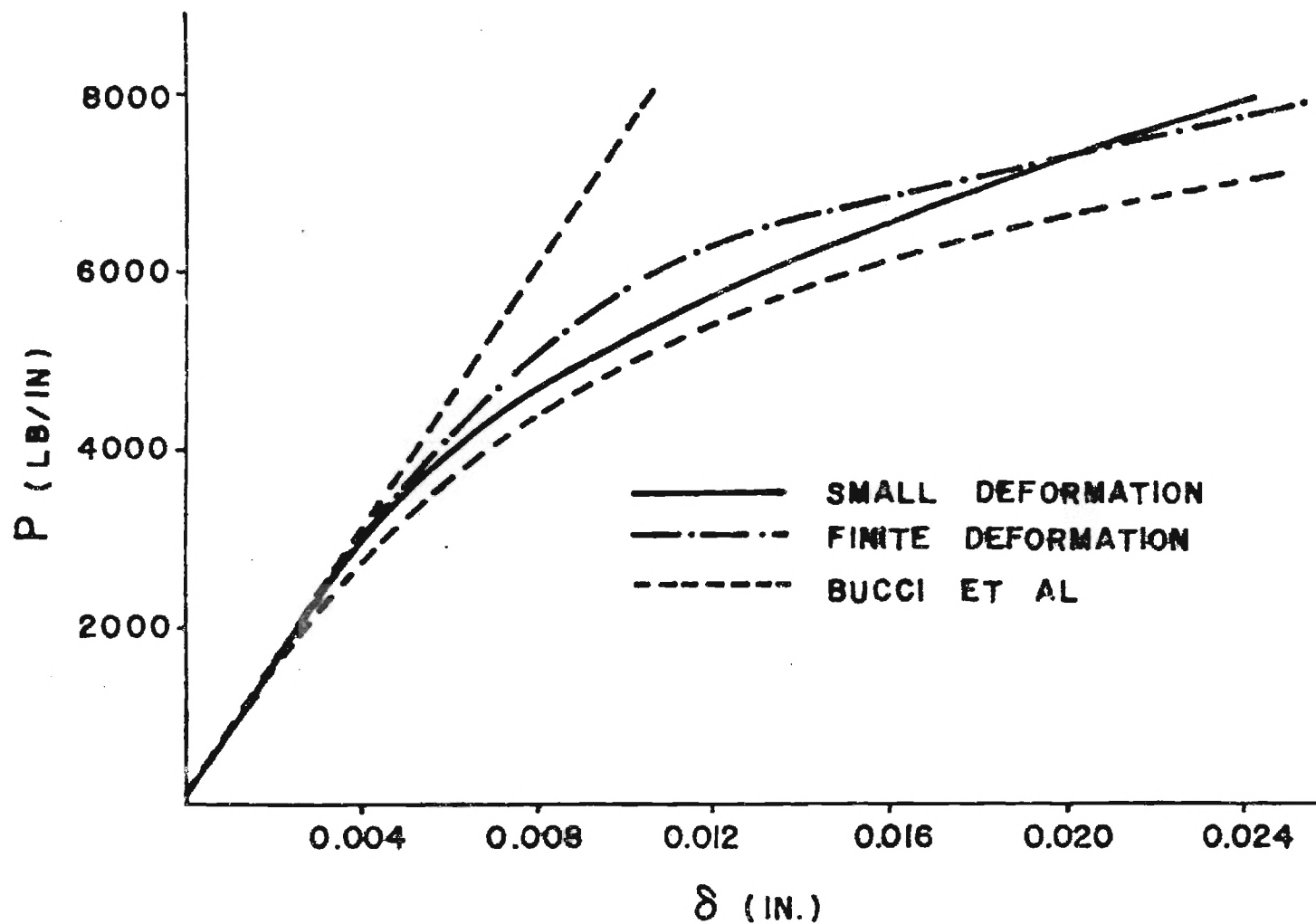


FIG. 4: Load vs. Load-Point Displacement for 3-Point Bend Bar

FIG. 11 (Taken from Ref. 16): Typical Load-Gauge Point Displacement Curves for Large-Scale Plastic Yielding Specimen.

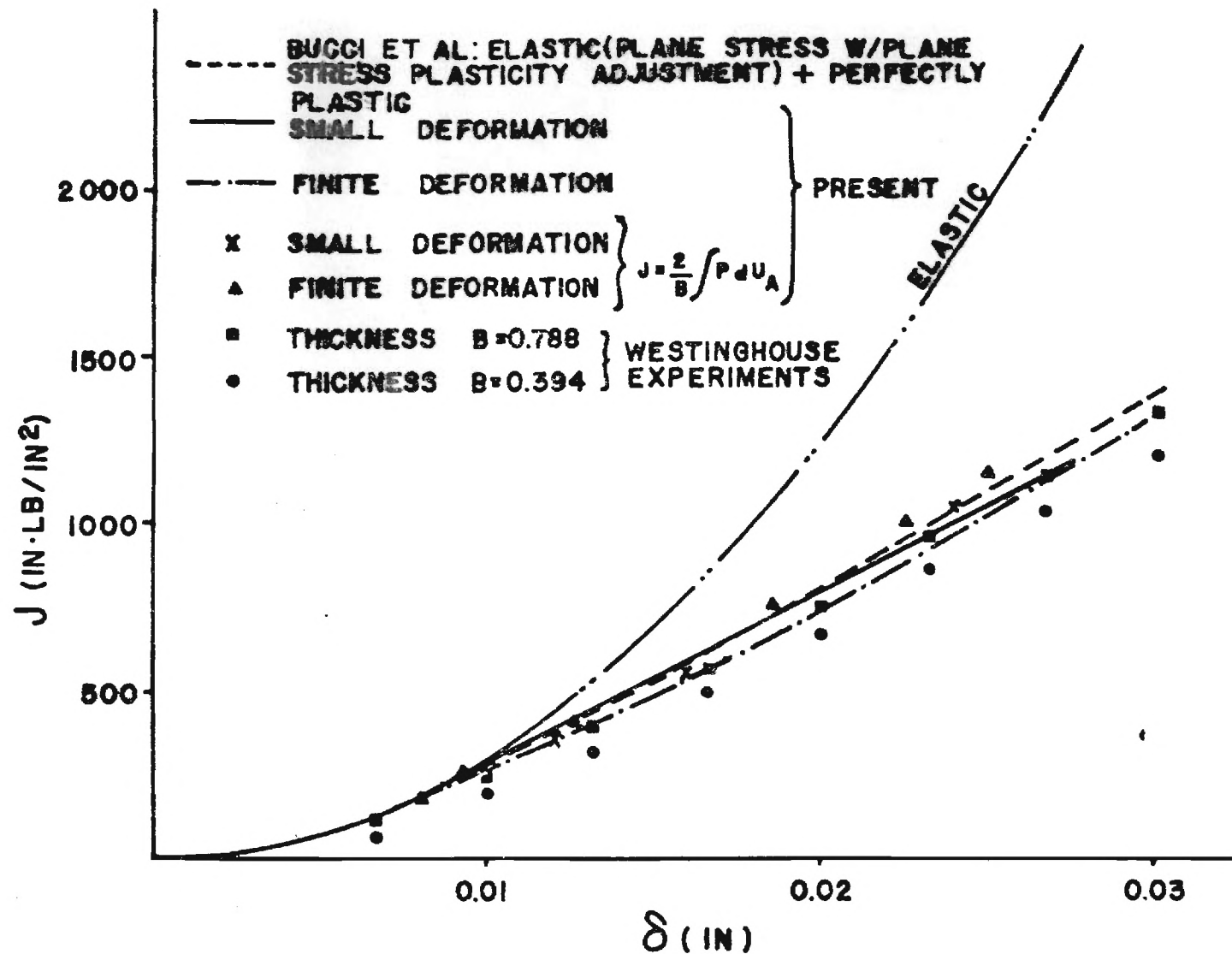


FIG.5: J-Integral vs. δ Curve for 3-Point Bend Bar

FIG. 12 (Taken from Ref. 16): Typical Results for J vs. Gauge Point Displacement: Comparison of Various Approximations with experimental data.

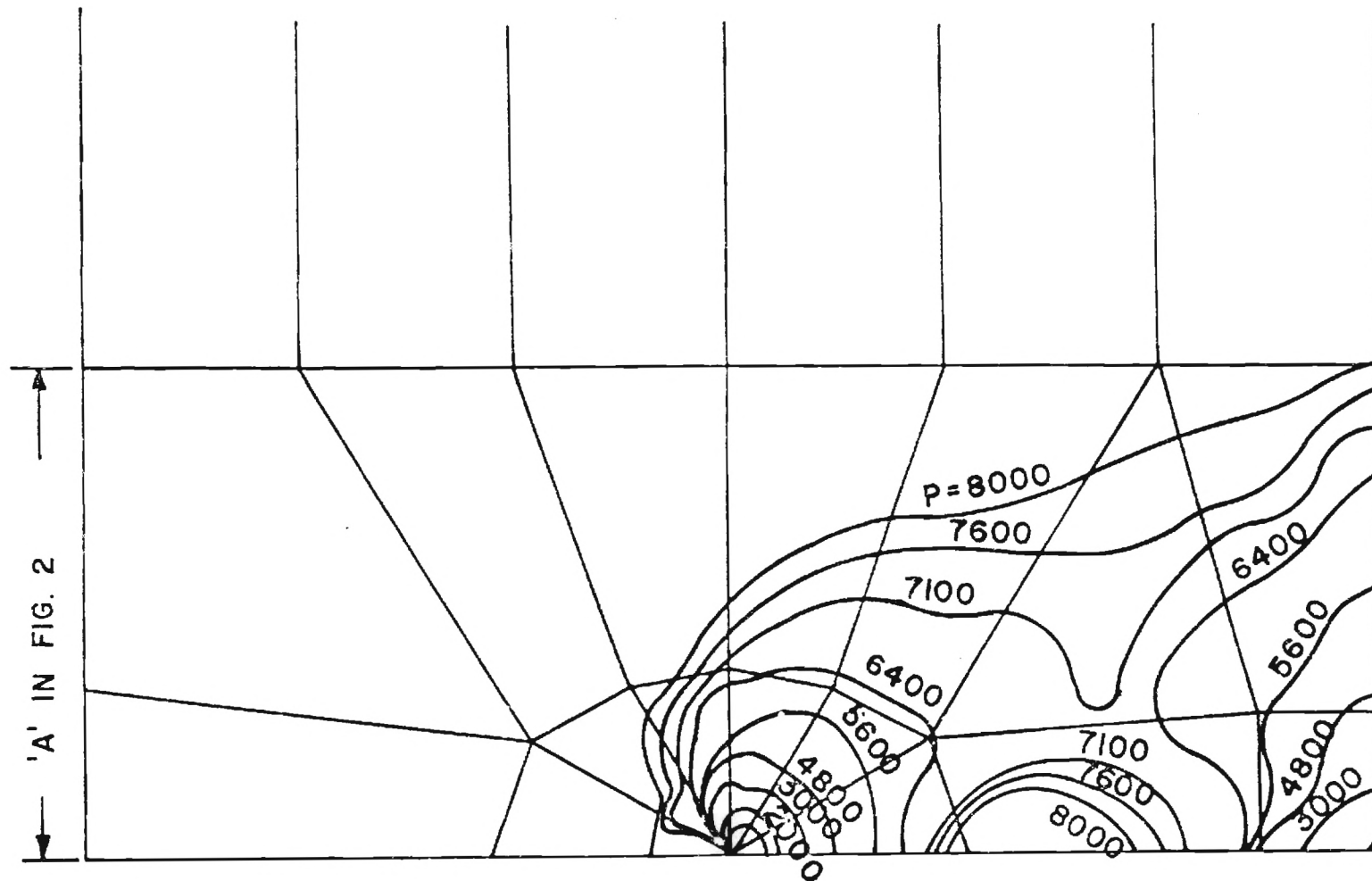


FIG.6: Yield-Zones at Various Load Levels
(Finite Deformation Analysis)

FIG. 13 (Taken From Ref. 18): Typical Yield zones in a Large-Scale Yielding Fracture Test Specimen.

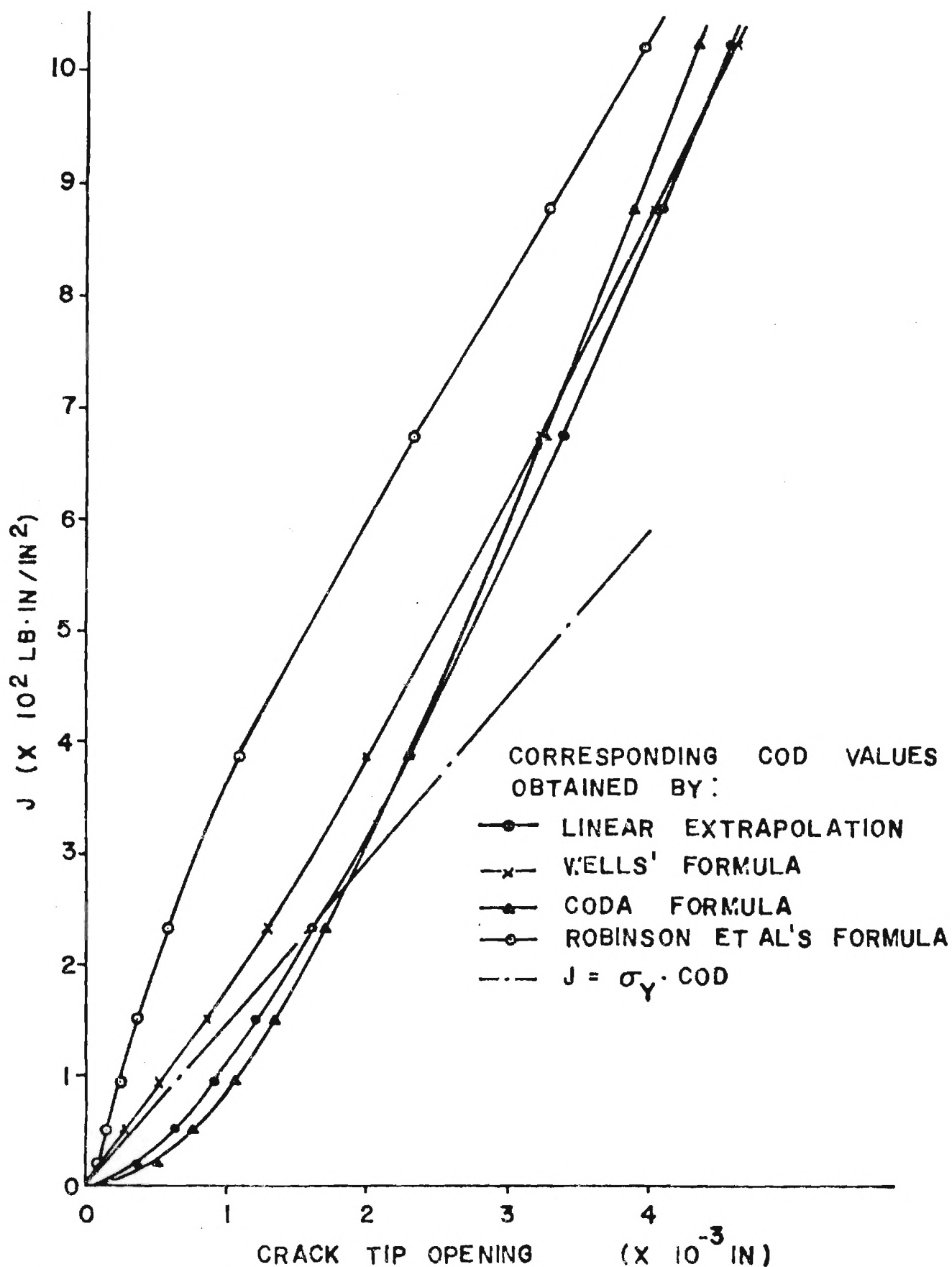


FIG.14 J vs COD Relation for a 3-Point Bend Bar.

FIG. 14 (Taken From Ref. 18): Correlation of Different Fracture-Initiation-Characterizing Parameters.

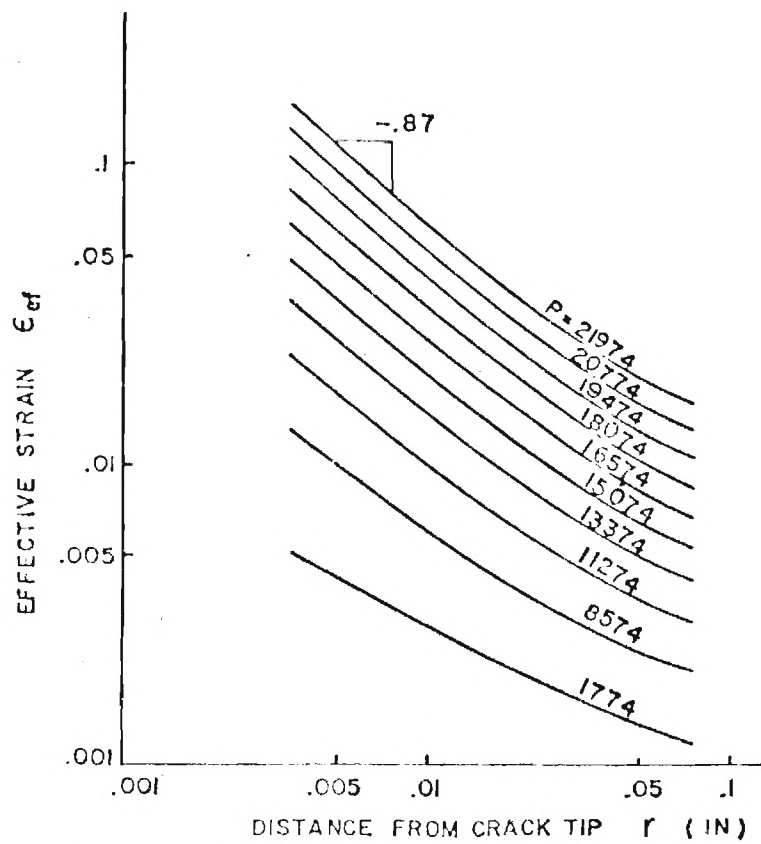


FIG. 11 BEHAVIOR OF STRAIN SINGULARITY NEAR THE CRACK TIP FOR A CT SPECIMEN.

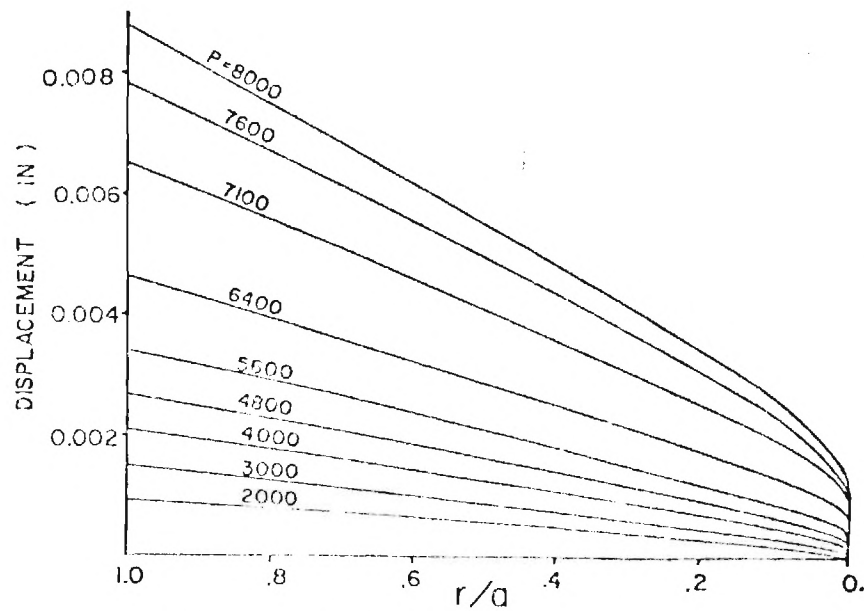


FIG. 12 CRACK-SURFACE DEFORMATION PROFILES AT VARIOUS LOAD LEVELS FOR 3-POINT BEND BAR.

FIG. 15 (Taken From Ref. 20): Nature of Near-field solutions at the crack-tip.

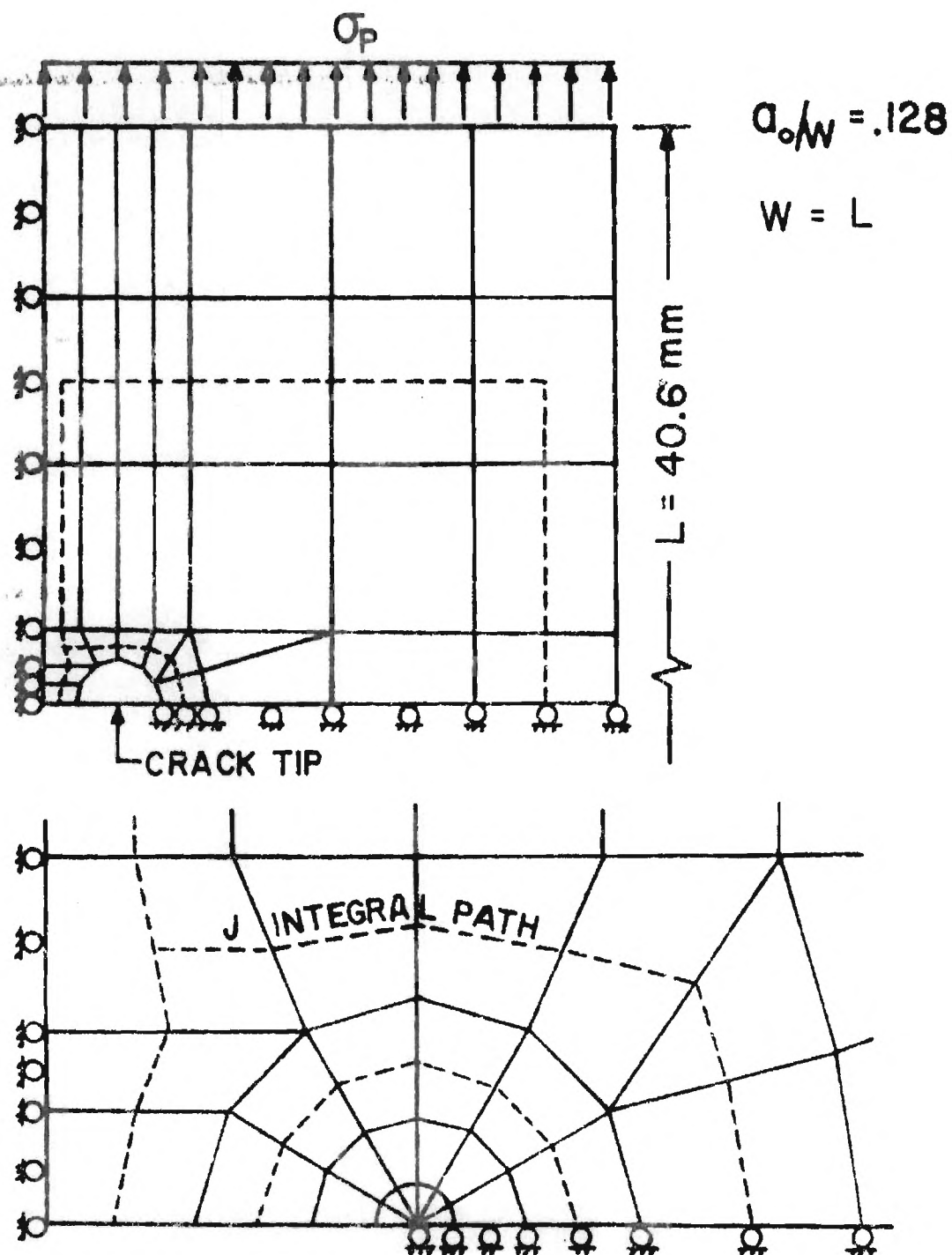


FIG 2. FINITE ELEMENT IDEALIZATION OF CENTER CRACKED SQUARE PLATE UNDER UNIAXIAL TENSION.

FIG. 16 (Taken from Ref. 30): A Typical Model Problem in Analysis of Stable Crack Growth.

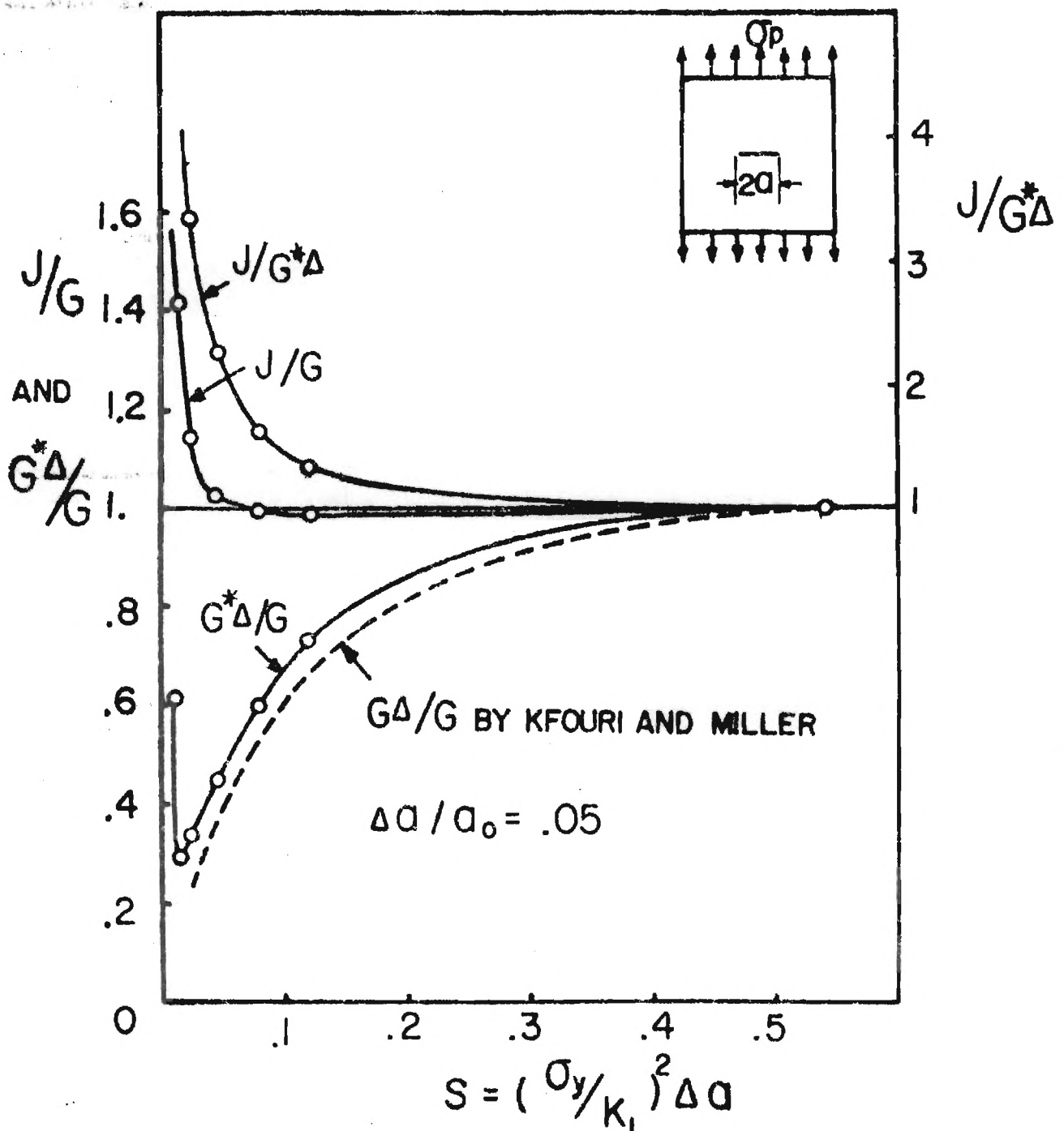


FIG 3. CRACK SEPARATION ENERGY RELEASE RATES: CONSTANT Δa AT VARIOUS LOAD LEVELS.

FIG. 17 (Taken From Ref. 29): Study of crack Separation Energy Release Rates in Ductile Materials: The parameter S is the ratio of Growth Step to the Plastic Yield zone size.

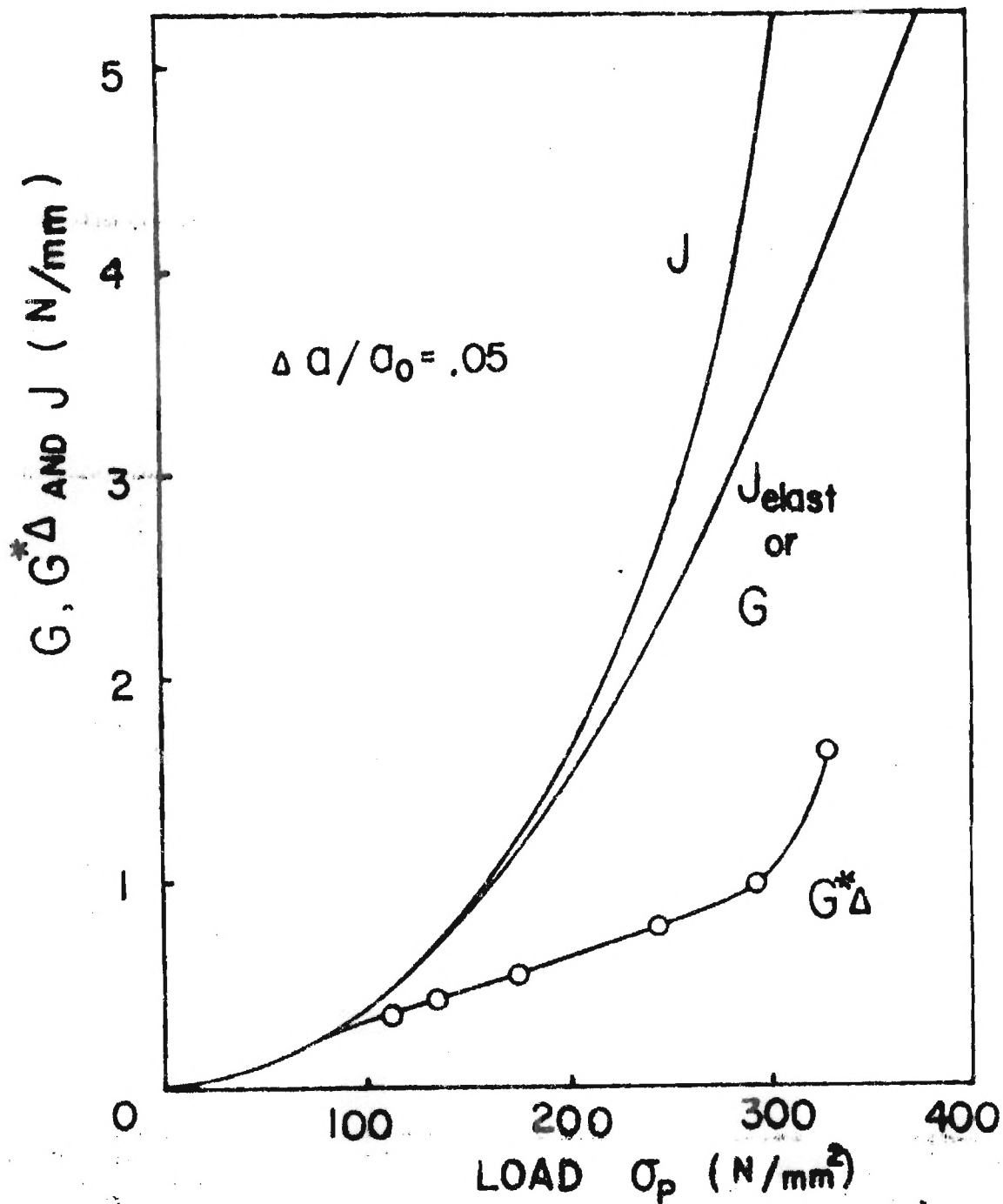


FIG 4. CRACK SEPARATION ENERGY RELEASE RATE:
CONSTANT Δa AT VARIOUS LOAD LEVELS.

FIG. 18 (Taken From Ref. 30): Further Elaboration of Results in Fig. 17.

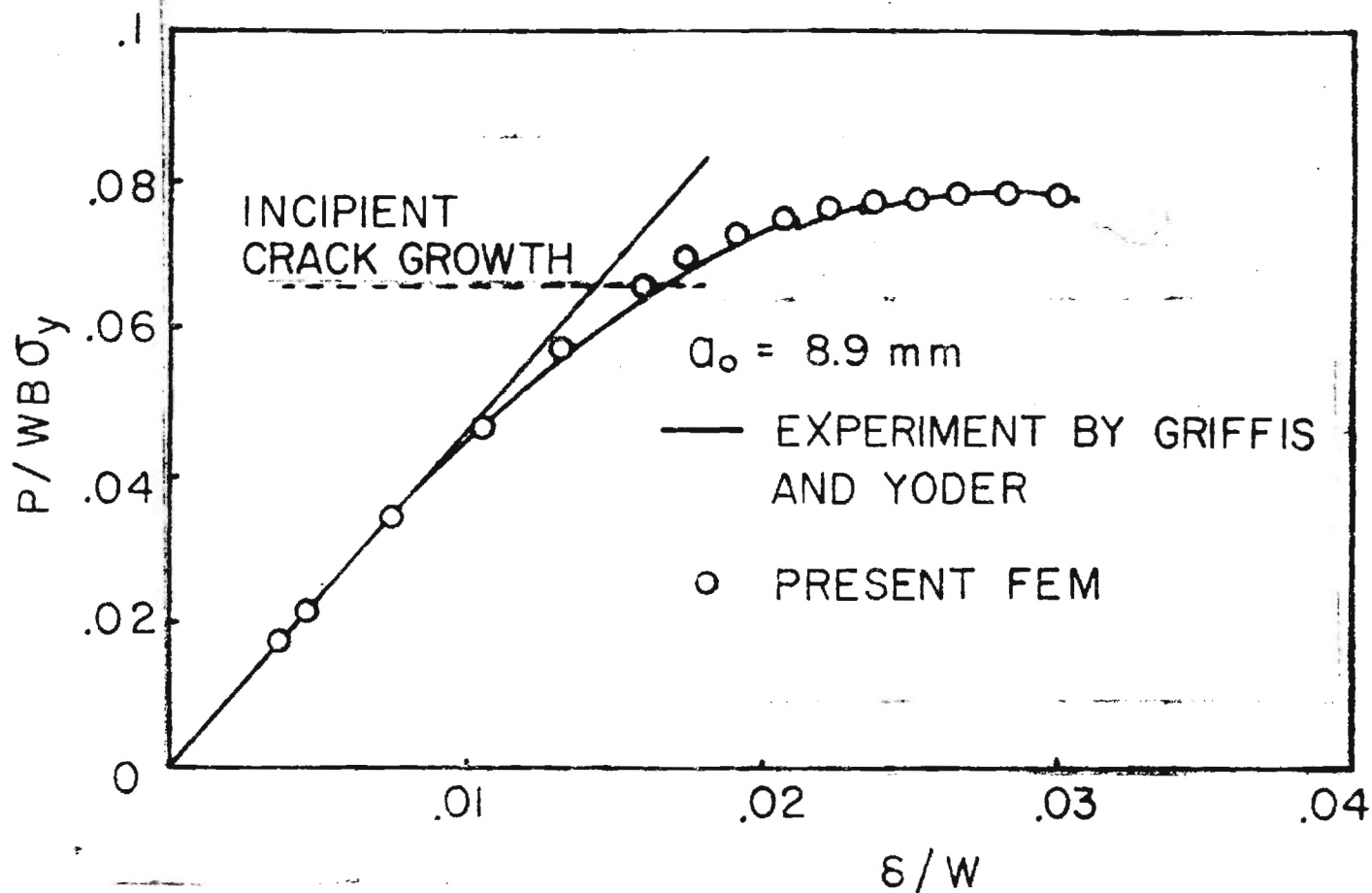


FIG 6. LOAD VS. LOAD POINT DISPLACEMENT
CRACK GROWTH CURVE.

FIG. 19 (Taken From Ref. 30): Results of Simulation of Experimental Stable Crack Growth Data.

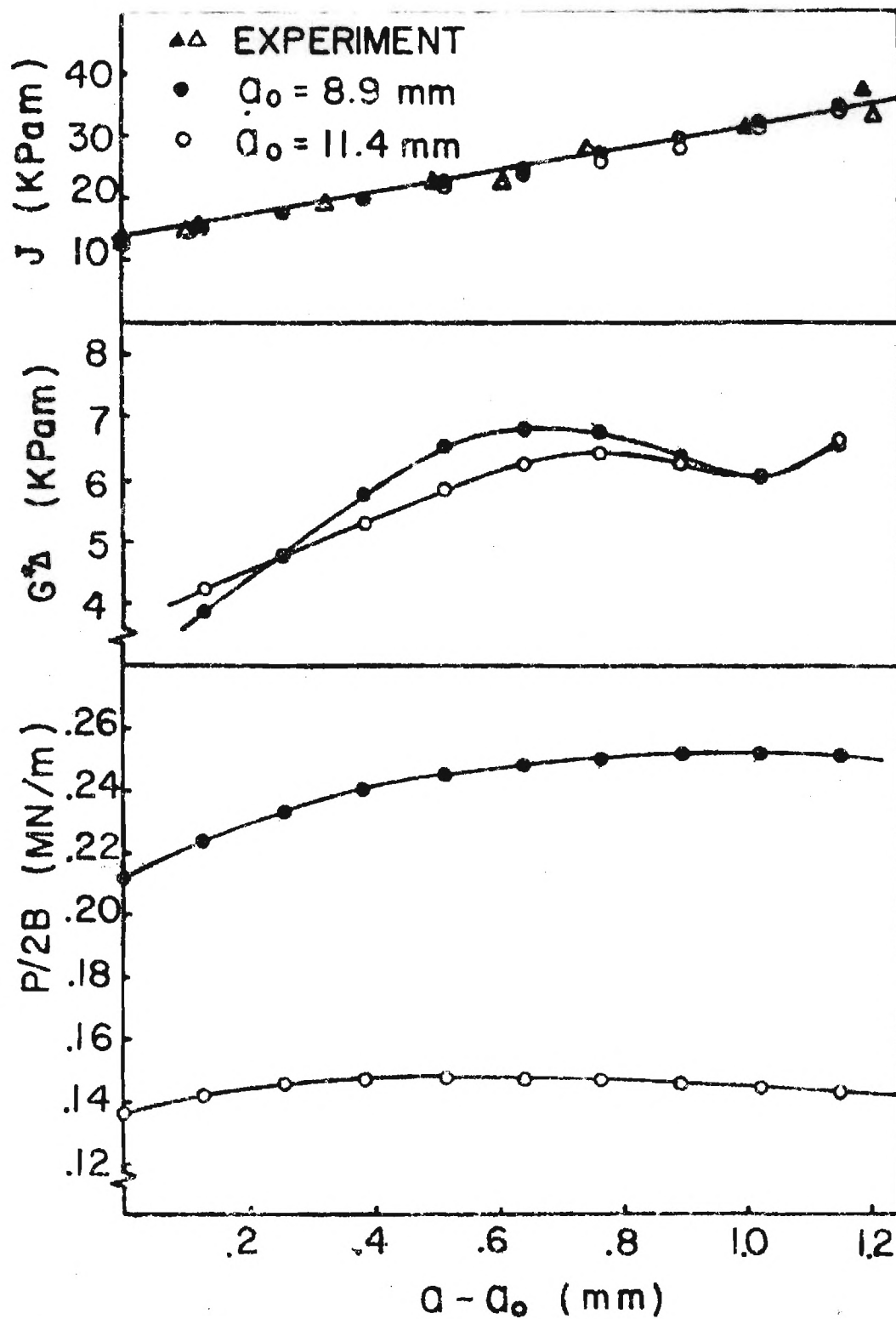


FIG 7. VARIATION OF J , $G^*Δ$, AND $P/2B$ DURING CRACK GROWTH.

FIG. 20 (Taken From Ref. 30): Results for Various Physical Parameters that may govern stability of Crack Growth, in the Simulation of Data in Fig. 19.

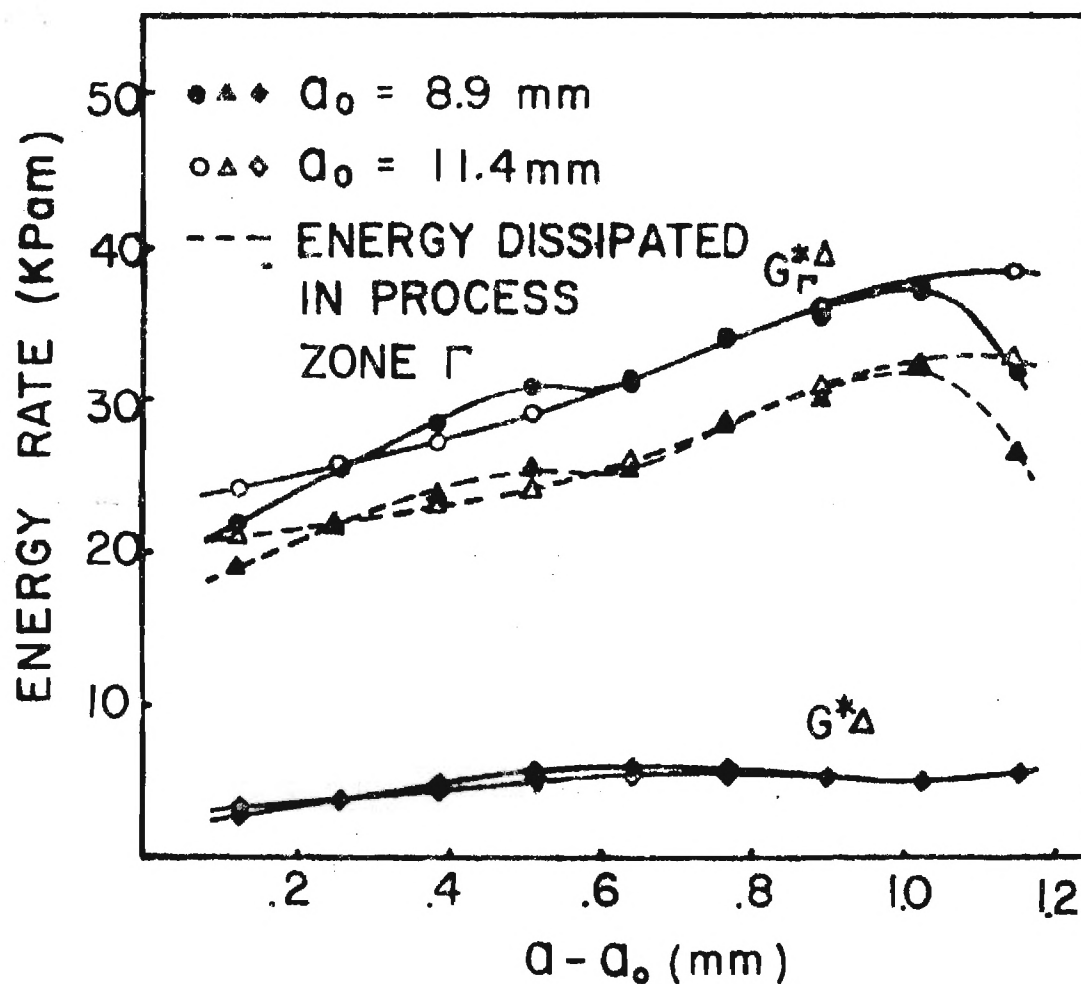
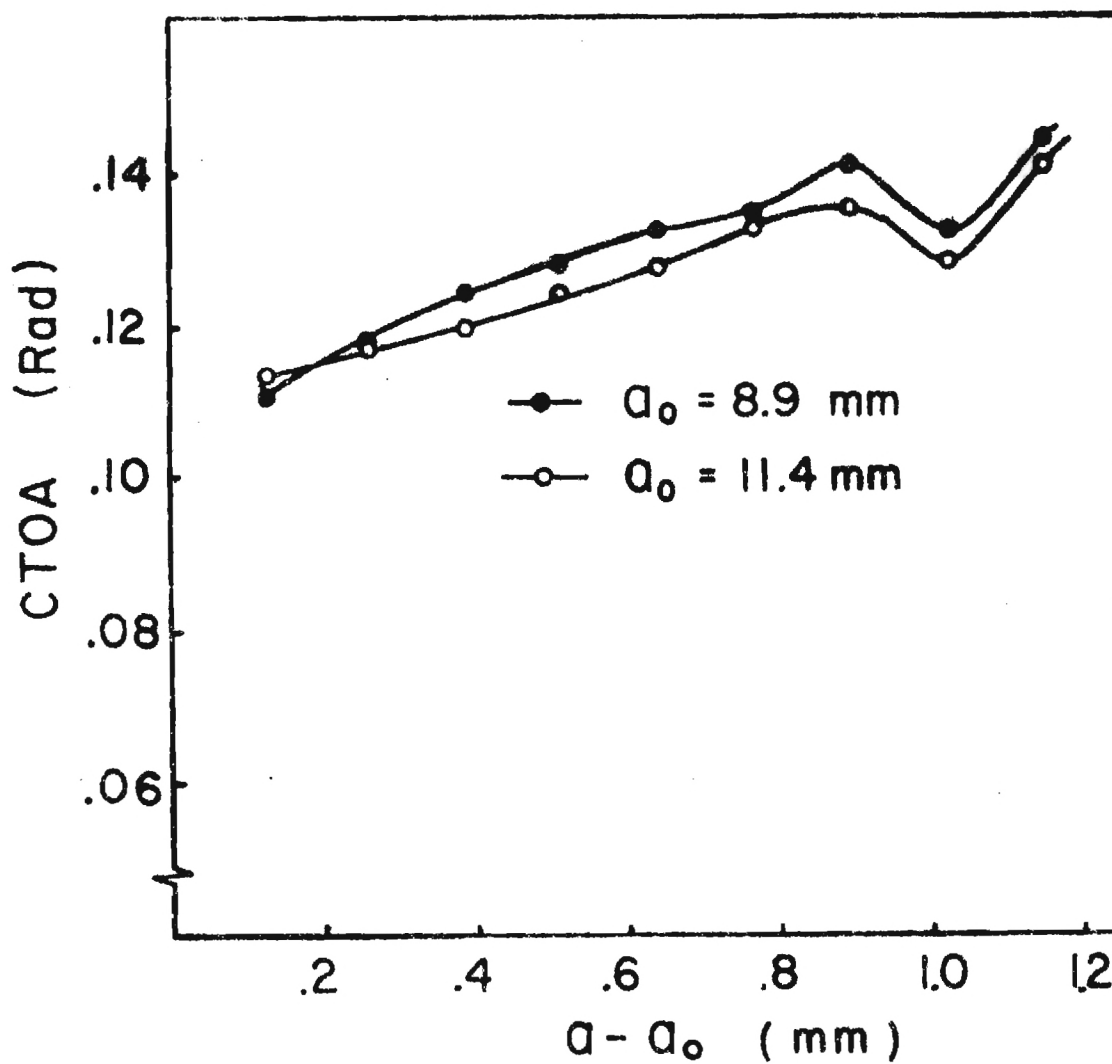


FIG 9. VARIATION OF $G_{\Gamma}^{*\Delta}$, $G^{*\Delta}$, AND ENERGY DISSIPATION IN PROCES ZONE DURING CRACK GROWTH.

FIG. 21 (Taken Fig. Ref. 30): Continuation of Results Similar to those in Fig. 19.



**FIG 10. VARIATION OF CRACK-TIP OPENING
ANGLE DURING CRACK GROWTH.**

FIG. 22 (Taken From Ref. 30): Continuation of Results Similar to those in Fig. 19.